Proceedings of the American Control Conference
Arlington, VA June 25-27, 2001

Tire Friction Modeling under Wet Road Conditions *

Xavier Claeys †, Jingang Yi‡, Luis Alvarez§, Roberto Horowitz¶, Carlos Canudas de Wit|| and Laurent Richard**

‡,¶ *Department of Mechanical Engineering, University of California, Berkeley, CA 94720-1740, USA*
§ *Instituo de Ingeniería Nacional Autónoma de México, 04510 Coyoacán DF, México*
|| *Laboratoire d'Automatique de Grenoble, UMR CNRS 5528, ENSIEG-INPG, ST. Martin d'Hres, France*
**,† *Direction de la Recherche RENAULT, 1. Av du Golf, 78288 Guyancourt, France*

Abstract

This paper reviews the physics of tire/road contact under wet conditions. A technique to include road moisture in physical tire models is proposed. Stationary-friction base tire models (see [2, 15, 5, 6]) are used to illustrate these developments.

1 Introduction

This paper addresses the influence of wet road conditions on the frictional characteristics between the tire tread and pavement. The results pertain to applications involving the design of new automatic vehicle features (e.g. brake control or road monitoring systems based on observers). There are many investigations reported in the literature that include useful interpretations concerning the development of tire models. Different road conditions are represented, in most cases, by several static mappings; each one of them describing a particular hygrometry level of the road (dry, wet, etc.). This technique is very simple but has serious limitations for simulation and control. A physical justification of the empirically derived mappings is not possible. These shortcomings are a major concern for the development of accurate simulation tools and the design of new braking and traction control algorithms.

In this paper we develop a new dynamic friction model for traction and braking control, which describes the tire/road interaction under wet conditions. Most current models do not give a comprehensive interpretation of the physics of the tire/road contact interface. In many cases the model focuses on the deformation properties of the tire, using empirical or physical relations (for a parametric approach see Pacejka or Burckhardt [11, 1, 4] or for finite element approach see [9]), but does not explain in detail the friction effects between the tire and road. As a consequence, wet road conditions that affect primary friction (through lubrication) are not easily included in the model. Secondly, many tire models are over-parametrized and do not include the physical geometric parameters as part of the large number of coefficients used to parameterize the model [11, 1, 9]. Therefore, it is difficult to include dependencies on external conditions inside these expressions such as the humidity of the road, which affects the geometry of the contact surface. Finally, it is also difficult to include external or internal perturbations in current models, since their underlying physics is not well understood.

A first step in the analysis of this problem was to develop a comprehensive description of the tire which is capable of separating different conditions that affect the dynamics of the tire/road interaction [6]. Hence, we identify three different aspects of a tire model: the friction between tire and road, the deformation of the tire shell, and the contact patch area. The contact interface is a consequence of the deformation of the shell although it changes affect the dry friction effective surface. In order to include new disturbances that affect the tire model, we have to analyze what are their effects on the different aspects of the tire (shell, friction forces, and contact geometry). Through this analysis, a clearer interpretation of the tire/road friction dynamics is possible because, in this case, we do not examine at the influence of a perturbation on the global model but rather its effect on a particular component.

This idea has been applied successfully in several instances. [7] separated the model into elementary blocks including tread groove depth of the tires and other phenomena such as pressure distribution. This model is interesting since it provides an initial description of the contact patch area as well as the pressure distribution. We believe this step is necessary for considering different road conditions, because the contact patch is the only part of the tire that is in real contact with the ground. It should be noted that the physical motivation given in previous papers is still limited. A similar study was performed in [9], where the patch surface variation with respect to tire shell deformation was determined, and the pressure distribution was also derived from a shell deformation finite element description. Unfortunately, the description was so complicated that this model is rarely used for control and cannot be easily extended to

*Research supported by California PATH grant MOU 373 and a gift from Renault SA.

†Renault Research Engineer; Xavier.Claeys@inpg.fr.
‡Graduate student; jgyi@me.berkeley.edu.
§Professor; alvar@pumas.iingen.unam.mx.
¶Professor; horowitz@me.berkeley.edu.
||Professor; canudas@lag.ensieg.inpg.fr.
**Renault Research Engineer; laurent.richard@renault.com

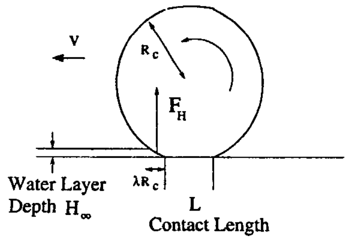

Figure 1: Schematic representation of tire under wet road

account for variations in the external conditions.

In this paper, we will discuss the inclusion in the model of road contamination by water. In section 2, a short review of the physics of water contamination will be presented. Then in section 3, a method for modeling these effects is proposed. Finally, in section 4 we illustrate this model using a stationary friction-based tire model. Simulation results for water contamination provides encouraging results. Concluding remarks are given in section 5.

2 Effect of water contamination

Road conditions have a tremendous effect on the tire behavior. The depth of water on the road greatly influences tire/road friction. These phenomena are complex and related to the different aspects of the contact patch between the tire and the road. Previous analysis made use of different water level categories such as thin or thick film of water on the road [3, 14, 13]. Even if the distinction between these two situations sounds artificial, it helps in understanding the contamination of the tire/road contact patch by water. In the limiting situation of a thin water layer, the contact between tire and road is completely lost due to full contamination of the interface *. This problem is called viscous hydroplaning [12, 10]. On the other hand, for a thick water layer, an extra force is generated in front of the tire due to the accumulation of water (hydro-dynamic forces). In this situation, the contact between the two surfaces could be lost without necessarily full contamination of the interface but mostly because of the hydro-dynamic force. This phenomenon is named dynamic hydro planning. The last case is more general than viscous hydroplaning due to the fact that other hydro-dynamical effects take place during the contact process.

Consequently, we can summarize these remarks as follows: for every set of experimental conditions in wet weather, there exists a definite velocity called the hydroplaning limit, beyond which hydro-planing occurs (see [10]). At this velocity, the wheel is in general supported by (see Fig. 1):

- A hydrodynamic upward thrust, just ahead of the contact area of the tire. The magnitude of this force is determined by the water layer depth h_∞ (for thick films) and vehicle velocity v. An expression for the upward thrust, F_H, can be found in [10]. It has been shown to depend upon v^2, where v is the velocity of the vehicle:

$$F_H = \kappa\lambda(h_\infty)v^2 F_N = Y_F F_N \qquad (1)$$

Here Y_F is a ratio, defined as a function of a constant κ, the normal load F_N, and a geometric angle λ, which is a function of the road water level h_∞:

$$\lambda = \sqrt{\left(\frac{L}{2R_c}\right)^2 - \left(\frac{h_\infty}{R_c}\right)^2 + \frac{2h_\infty}{R_c}\sqrt{1-\left(\frac{L}{2R_c}\right)^2}} - \left(\frac{L}{2R_c}\right) \qquad (2)$$

Where L is the nominal contact patch length, and R_c is the free radius of the tire.

- A squeezed film and wiping effect in the contact area. The squeezed film effect is due to the inability of the tread elements traversing the contact area to remove the last few tenths of a millimeter of water in the available time. The wiping effect is due to the longitudinal and lateral relative motion of the tire elements in the contact area.

Remark 2.1 *F_H approaches the load of the wheel at high velocities (over 200 km/h, which is not realistic) depending of the amount of water h_∞ on the road. Thus, as stated before, the hydrodynamic thrust F_H is not the only factor contributing to hydro-planing. This discrepancy is due to variations in the contact area due to a thin film of water squeezed at the tire/road interface: the thin water layer supports a fraction of the load of the wheel, and hence the effective contact area and the effective normal load decreases.* ○

The inclusion of all of these effects in the tire model is possible if we can develop dynamic expressions for the contact patch and the effective load of the tire, based on physic laws. Determining the contact patch is a difficult problem, and several empirical or semi empirical approaches have been proposed in the literature [8, 12]. In these references, the road/tire interface (contact area) is divided into three areas, where the major effects are different: dry contact, micro-water layer region, and macro-water layer region. A simple model can then be obtained to scale the effective contact area, given certain drainage characteristics of the ground and the tire.

These discontinuous interpretation of the three areas is not satisfactory for simulation or control purposes, because this requires a significant amount of characterizations and experimental measurements. In the next section we will propose a more sophisticated interpretation of the contact patch dynamics based on fluid mechanics to solve this problem.

*When water has invaded the complete contact area between the tire and the road

2.1 Contact surface variation models

A preliminary version of this technique was presented in [10]; further developments are given in [13]. This technique is very promising but has to be simplified for use in actual tire models. We first present some basic concepts from the literature and then study one example.

The main idea in [10] is to identify the dynamics of the sinkage of the tire when a thin film of water is present at the tire/road interface. The fluid at the interface is modeled as viscous, and the sinkage dynamics of the tire can be determined using for example the Navier-Stokes equations. At the time these studies were performed, computers were not sufficiently powerful, thus simple analytic solutions were developed. A numerical solution was also derived in [13] using some simplifications.

The effective tire/road contact surface is a function of the time taken by the tire to achieve contact with the ground. As a consequence, all the parameter calculations are based on the water clearance time or time of sinkage. This time will vary for different wheel speeds, tread stiffness, ground pattern characteristics, etc.

Definition 2.1 The sinkage time of the tire Δt_2 is defined as the time for the water level to decrease from its initial height h_0 (h_0 is the initial height of water in the squeeze film area) to a final height h_{min}, where h_{min} is small enough so that we can consider that the tire and road surfaces are in contact. •

The time for a portion of the tire to cross the contact patch is given by $\Delta t_1 = \frac{L}{R_c w}$, where L is the footprint length, w the wheel angular velocity, and R_c was defined earlier as the tire free radius.

If $\Delta t_1 > \Delta t_2$, the water level is not sufficient to completely prevent dry contact and the portion of the tire in dry contact has length

$$L^{water} = R_c w(\Delta t_1 - \Delta t_2) = Y_R L, \qquad (3)$$

with Y_R being the fraction of the tire that is in real contact with the ground.

A simple method for solving this problem uses the analogy between the behavior of the tire in the sinkage zone, or squeeze film area, and the sinkage of a flat plate over a randomly rough surface area. Considering the sinkage of a flat plate over a rough surface, there is a bulk flow of escaping fluid between the plate and the asperity tips, and an open channel flow between asperities (called channelization). As the plate approaches the peaks of the asperities, there is virtually no bulk flow, and the channel flow becomes closed because the plate provides an upper boundary in this position. This approach gives a good intuitive understanding of the problem (see [10]), but it is still very crude and we are currently looking for a better model. It appears that describing each tread element individually using viscous flow theory may provide such a model. Two solutions are important in the analysis of viscous flow at the interface:

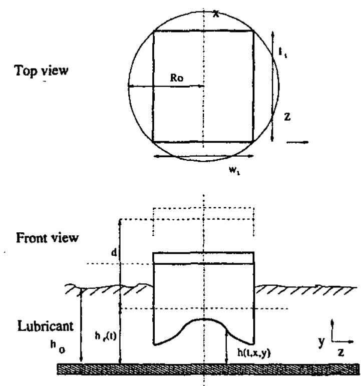

Figure 2: Film of water between a tread block and the ground, with h_0 being the water film thickness, d the tread element thickness, $h_r(t)$ the mean position of the tread element (assuming the tread is rigid) and $h(t, x, y)$ the position of the tread element (assuming that the tread is flexible)

a) Time varying solution: The Navier Stokes equation for the flow is a partial differential equation. Identifying the time dynamics of the water level under the tire is difficult, since many parameters such as the tire tread stiffness, speed, etc. affect these dynamics. Several approximations are necessary to obtain an analytical solution to the problem. In this paper, we will assume that the tire tread elements are **rigid**. Then, the Reynolds equation becomes a simple time varying equation. The solution of this equation is important to evaluate whether water is removed quickly enough from the contact patch, in order to achieve dry friction.

b) Stationary solution: Finding a time varying solution to the problem is not useful if one wishes to apply this theory to an actual tire model, since we want to determine the time required by a tread element to achieve contact with the ground and relate this quantity to the effective contact surface. Thus, as an alternative we will tune a static model with respect to the road conditions. This solution is still quite simple since several dynamic effects have not yet been included in the problem (slip dynamics, tread flexibility, etc.).

To derive a stationary solution for a circular tread element shown in top of Fig. 2), we will study the following governing equation for the descent of the tread element through a thin film of water:

$$\frac{1}{R}\frac{\partial}{\partial R}(H^3 R\frac{\partial P}{\partial R}) = \frac{1}{\pi}\frac{dH}{dt}. \qquad (4)$$

$R = \frac{r}{R_0}$ is a normalized radius where r is the radial coordinate and R_0 is the tread radius. $H = \frac{h_r}{h_0}$ is the normalized water height, where h_0 is the initial height of water in the squeeze film area (for simplicity we take $h_0 = h_\infty/100$). $P = \frac{p}{P_{ref}}$ is the normalized pressure,

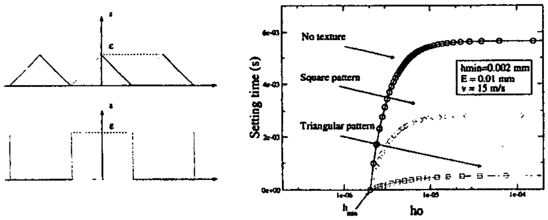

Figure 3: Effect of the pattern design on the setting time. For example, the setting time is smaller for a triangular pattern than for a squared pattern for the same texture amplitude. On the other hand no texture at all implies a larger setting time.

where p is the tread pressure and $P_{ref} = \frac{12\mu V_{ref} R_0^2}{h_0^3}$. V_{ref} is a reference velocity and μ is the fluid viscosity.

The Reynolds Eq. (4) written in *dimensionless* form is valid only for a thin film of water ($h_0 < 0.2mm$). However if we assume that downward water inertia is removed in the hydrodynamic region of the upward thrust, this part of model describes only the thin squeezed film of water $h_r(t)$ remaining at the tire/road interface.

The effect of slip has been neglected in this equation, but can be added if necessary. However, in most cases, it turns out to be negligible. Since we are interested in the load $W(t)$ that the fluid supports at each instant of time, with $W(T) = 2\pi \int_0^1 P(t,r)RdR = \pi P$, we may integrate Eq. (4) to get

$$W(t) = -\frac{1}{2}\frac{dH}{dt}\int_0^1 \frac{R^3 dR}{H^3}, \tag{5}$$

with H considered as a constant at each instant of time. Eq. (5) reduces to

$$W(t) = -\frac{1}{8}\frac{dH}{dt}\frac{1}{H^3} \tag{6}$$

This equation can be solved for a simple time varying function $H(T)$ where $T = \frac{tV_{ref}}{h_0}$ is the normalized time to obtain

$$H(T) = \frac{1}{\sqrt{1+I(T)}}, \tag{7}$$

with $I(T) = A\int_0^T W(\tau)d\tau$ and $A = 16$. Notice that the constant A in this expression will be different for different tread element geometries (for example. $A = 18.1$ for a square element instead of 16 for a round element). Thus, the above results are valid for other geometries.

We have described the descending dynamics of the tread in the fluid (in dimensionless form). Suppose that there is contact for $H < H_{min}$, where H_{min} is a given normalized limit spacing between the tire and the road ($H_{min} = \frac{h_{min}}{h_0}$ and h_{min} is the minimum spacing between tire and road). Then, we can determine the setting time from the solution given in Eq. (7). Supposing that the pressure distribution is uniform and constant, we obtain this result in a non-normalized form

$$\Delta t_2 = \frac{12\mu R_0^2}{A\pi p}\left[\frac{1}{h_{min}^2} - \frac{1}{h_0^2}\right]. \tag{8}$$

Using this expression and definition 2.1, we can find Y_R in Eq. (3), the fraction of the tire in contact with the road.

Using the same method, it is possible to include the effect of the ground surface. This task is however more difficult and it is helpful to consider the work developed in [13]. A similar equation to (4) can be written if the ground texture is isotropic and a solution similar to Eq. (8) is obtained. In this case the setting time Δt_2 is a function of amplitude ϵ of the road texture. The texture density ρ will modify the mathematical expression of Δt_2. For a triangular road texture,

$$\Delta t_2 = \frac{12\mu R_0^2}{A\pi p}\left[\frac{1}{h_{min}^2 - \epsilon^2} - \frac{1}{h_0^2 - \epsilon^2}\right] \tag{9}$$

For three different textures, the setting time is given as a function of the water level in the transition zone h_o (Fig. 3). We observe that the initial water level in the transition zone h_0 does not affect greatly the setting time. An interesting result is that the effective fraction of the tire in contact with the ground (Y_R) is a function of both the texture amplitude ϵ, and the angular wheel velocity w. In other words, for a given level of water on the road h_∞ and a fixed travel velocity v, the hydroplaning limit (or the adhesion characteristic) will vary only as a function of the road surface properties (texture and density) and the wheel acceleration input. Notice that in this example we have neglected the tread deformations, which could interfere with this process.

Using this result, we can now describe the tire behavior in wet road conditions as a function of the external condition (h_∞), the internal state (w, v and F_N) and the road properties (ϵ, ρ). Keeping track of the road texture properties will become an important issue in tire/road friction modeling and estimation under wet road conditions.

2.2 Preliminary conclusions

The following conclusions can be obtained from the contact interface model that has been discussed.

• In the case of a uniform tire pressure distribution, the normal force resulting from the load of the vehicle depends on the ratios Y_R and Y_F defined in previous sections (see Eqs. (1) and (3)):

$$F^{water} = \int_{-L/2}^{L/2} Pwd\xi - \int_0^L P^{water}wd\xi = (Y_R - Y_F)F_N = Y_L F_N \tag{10}$$

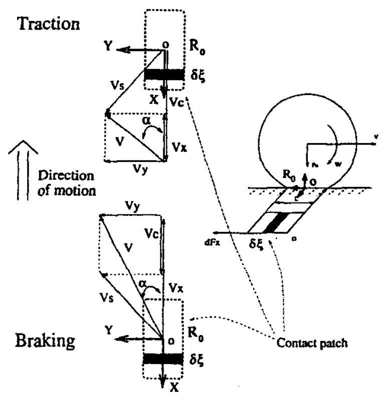

Figure 4: Convention for slip definition. Two cases are considered: braking and traction

• The contact patch length has an expression which depends on the given ratio Y_R as well:

$$L^{water} = Y_R L \tag{11}$$

3 Application to tire models

The friction-based tire models presented in [5, 2, 15] have been shown to reproduce the classic stationary friction coefficient vs. slip ratio curves known as the "Magic Formula". We will use in this paper a stationary version of the derived tire model extended to three dimensions (see [6]) and discuss the consequences of previous results.

3.1 Definition of slip

Most tire/road friction models are based on the important observation that a tire is slipping while rolling on a road surface. The slip has been used as a natural phenomenon to parameterize tire models.

Fig. 4 illustrates the notation that will be used in this paper. V_R is the speed of the moving frame given by $V_R = V$, where V is the vehicle speed (the frame moves with the tire). V_c is the circumferential velocity of the tire given by $V_c = R_c w$, where R_c is the "free" radius of the tire, w is the wheel angular velocity, and α is the slip angle. Finally, V_x and V_y are the two components of the frame speed V_R, and V_{sx} and V_{sy} are the two components of the slip vector V_s defined by $V_s = \sqrt{(V_x - V_c)^2 + V_y^2}$. As slipping occurs in both in longitudinal and lateral directions, two relations can be defined for lateral and longitudinal slip. We also use two conventions to distinguish the braking and traction cases (see Fig. 4), so that the slip is well defined even when the wheel speed becomes zero.

• In the braking case, longitudinal slip S_{s_b} and lateral slip S_{α_b} are given respectively by

$$S_{s_b} = \frac{V_x - V_c}{V_x} \tag{12}$$

$$S_{\alpha_b} = |\tan \alpha| \tag{13}$$

Since $V_x > V_c$ and $V_x \neq 0$ during braking, $S_{s_b} > 0$.

• In the traction case, longitudinal slip S_{s_t} and lateral slip S_{α_t} are respectively given by

$$S_{s_t} = \frac{V_c - V_x}{V_c} \tag{14}$$

$$S_{\alpha_t} = (1 - S_s)|\tan \alpha| \tag{15}$$

Since $V_c > V_x$ and $w \neq 0$ during traction, $S_{s_t} \geq 0$.

3.2 Three dimensional friction tire model

Finally, we include the road condition given by Eqs. (10) and (11) in the model. The extended stationary model in traction is given by the following expression:

$$\frac{F_x}{F_n} = -Y_L \gamma_x(V_{sx}) \mathrm{sign}(V_{sx}) g_x(V_{sx}) \left[1 + \frac{g_x(V_{sx})}{Y_R L \sigma_{0_x} S_{s_t}} \left(e^{-\frac{\sigma_{0_x} Y_R L S_{s_t}}{g_x(V_{sx})}} - 1\right)\right] + Y_L(\sigma_{1_x} + \sigma_{2_x}) V_{sx} \tag{16a}$$

$$\frac{F_y}{F_n} = -Y_L \gamma_y(V_{sy}) \mathrm{sign}(V_{sy}) g_y(V_{sy}) \left[1 + \frac{g_y(V_{sy})}{Y_R L \sigma_{0_y} S_{\alpha_t}} \left(e^{-\frac{\sigma_{0_y} Y_R L S_{\alpha_t}}{g_y(V_{sy})}} - 1\right)\right] + Y_L(\sigma_{1_y} + \sigma_{2_y}) V_{sy} \tag{16b}$$

$$\frac{M_z}{F_n} = -Y_L \frac{\gamma_y(V_{sy}) \mathrm{sign}(V_{sy}) g_y^2(V_{sy})}{2\sigma_{0_y}^2 S_{\alpha_t}} \left[\left(e^{-\frac{\sigma_{0_y} Y_R L S_{\alpha_t}}{g_y(V_{sy})}} - 1\right) + \left(\frac{2 g_y(V_{sy})}{Y_R L \sigma_{0_y} S_{\alpha_t}}\right)\left(e^{-\frac{\sigma_{0_y} Y_R L S_{\alpha_t}}{g_y(V_{sy})}} - 1\right)\right] \tag{16c}$$

Where γ_i are two functions defined as in [5, 6] for the lateral and longitudinal cases, and σ_{i_j}, g_i are the classical parameters of the LuGre dry friction model, for the lateral and longitudinal directions ($i = 1, 2, 3$ and $j = x, y$). For the braking case a similar formula can be derived with $S_{s_t} = \frac{S_{s_b}}{1 - S_{s_b}}$ and $S_{\alpha_t} = \frac{S_{\alpha_b}}{1 - S_{s_b}}$.

4 Numerical example

The results of a numerical simulation are illustrated in Figs. 5 and 6, for a typical tire referenced as 165-65R14. The model was first tuned to fit the classical "Magic Formula" on a dry surface, and then it was simulated for different water films and various texture amplitudes. Fig. 5 illustrates the hydro-dynamic properties of the tire under wet road conditions. As predicted, increasing the thickness of the water on the road h_∞ for a given travel velocity v dramatically reduces the resulting friction. Furthermore, Fig. 6 shows the effect of varying the texture amplitude ϵ on the friction characteristics. In this case, the water layer h_∞ is given and the road pattern is specified as triangular. As ϵ, is varied the setting time is slightly affected. We observe that friction increases as texture amplitude increases.

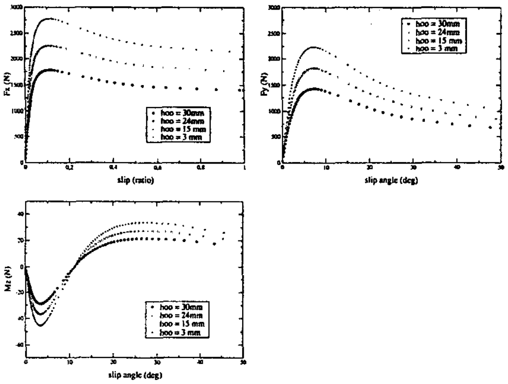

Figure 5: Effect of water depth h_{∞} on a stationary tire model ($v = 25m/s$ and triangular texture of amplitude $\epsilon = 0.06mm$)

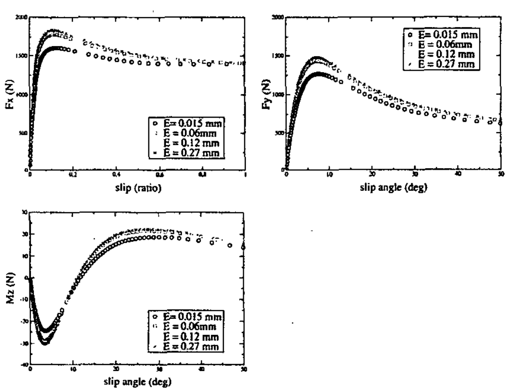

Figure 6: Effect of road texture amplitude E on a stationary tire model (texture: triangular motif, $v = 25m/s$ and $h_{\infty} = 30mm$)

5 Conclusion

An approach to include variations in road moisture in a tire/road friction model has been described in this paper. In particular, both the hydrodynamic and lubrication effects of the contact zone have been discussed. Different road textures and densities can be introduced in friction based tire models (LuGre) using relatively simple methods. It is known that the tread stiffness characteristic affects the sinkage time and this point should be investigated more carefully. Furthermore, a technique to decouple the effects of the road pavement from those of the tire under wet conditions has been proposed, and could lead to an important breakthrough for embedded road or vehicle diagnostic tools.

6 Acknowledgments

The authors wish to express their appreciation to A. Richard from Renault SA. for encouragements and valuable discussions. The authors would also like to acknowledge the support of Renault Research Division who initiate this research.

References

[1] Egbert Bakker, Lars Nyborg, and Hans B. Pacejka. Tyre Modeling for Use in Vehicle Dynamics Studies. *SAE Transactions*, (870421):190–204, 1987.

[2] P.A. Bliman, T. Bonald, and M. Sorine. Hysteresis Operators and Tyre Friction Models: Application to Vehicle Dynamic Simulation. In *Proceedings of ICIAM'95*, Hamburg, Germany, 1995.

[3] A. Browne and D. Whicker. Design of tire tread elements for optimum thin film wet traction. *SAE paper*, (770278), 1977.

[4] M. Burckhardt. *ABS und ASR, Sicherheitsrelevantes, Radschlupf-Regel System. Lecture Scriptum*, University of Braunschweig, Germany, 1987.

[5] Carlos Canudas de Wit and Panagiotis Tsiotras. Dynamic Tire Friction Models for Vehicle Traction Control. In *Proceedings of* 38^{th} *IEEE Conference of Decision and Control*, Phoenix, AZ, 1999.

[6] X. Claeys, J. Yi, L. Alvarez, R. Horowitz, C. Canudas de Wit, and L. Richard. A Simple 3D Parametric Dynamic Tire/Road Friction Model for Vehicle Simulation and Control. *Submitted to the 2001 ASME-IMECE*, 2001.

[7] F. Fancher, J. Bernard, C. Clover, and C. Winkler. Representing truck tire characteristics in simulations of braking and braking in turn maneuvers. In *Supplement to Vehicle System Dynamics: Tyre Model For Vehicle Analysis*, volume 27, 1997.

[8] W. Horne and F. Buhlmann. A method for rating the skid resistance of micro/macrotexture characteristics of wet pavements. In *Frictional Interaction of Tire and Pavement, American Society for Testing and Materials*, pages 191–218, 1983. STP 793.

[9] G. Mastinu, S. Gaiazzi, F. Montanaro, and D. Pirola. A semi-analytical tyre model for steady- and transient state simulations. In *Supplement to Vehicle System Dynamics: Tyre Model For Vehicle Analysis*, volume 27, 1997.

[10] D. Moore. Drainage criteria for runway surface roughness. In *Royal Aeronautical Society*, 1964.

[11] H. Pacejka and I. Besselink. Magic formula tire model with transient properties. In *Supplement to Vehicle System Dynamics: Tyre Model For Vehicle Analysis*, volume 27, 1997.

[12] M. Pottinger and T. Yager. *The Tire Pavement Interface.* ASTM Publication, 1985. ASTM Publication Code Number 04-929000-27.

[13] S. Rohde. On the effect of pavement microtexture on thin film traction. *International Journal of Mechanical Science*, (18):95–101, 1975.

[14] S. Rohde. On the combined effects of tread element flexibility and pavement microtexture on thin wet traction. *SAE paper*, (770277), 1977.

[15] M. Sorine and J. Szymanski. A new all-vehicle-speed dynamic tire model. In *IFAC Symposium on Control in Transportation Systems 2000*, Braunschweig, Germany, June 13-15 2000.