AHS Safe Control Laws for Platoon Leaders

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***Abstract*—The AHS architecture of the California PATH program organizes traffic into platoons of closely spaced vehicles. A large relative motion between platoons can increase the risk of high relative velocity collisions. This is particularly true whenever platoons are formed or broken up by the join and split control maneuvers and by the decelerate to change lane control maneuver, which allows a platoon to create a gap before switching from one lane to another. In this paper we derive a safety region for the relative velocity between two platoons. By guaranteeing that the relative velocity between platoons remains in this region, impacts of high relative velocity can be avoided. Under normal operating conditions, there are four control laws for a platoon leader: leader law, join law, split law, and decelerate to change lane law. For each control law, a desired velocity profile for the platoon that satisfies safety and time-optimality requirements is derived. A nonlinear velocity controller is designed to track the desired velocity profile within a given error bound. When safety is not compromised, this controller keeps the acceleration and jerk of the vehicles in the platoon within comfort limits.**

***Index Terms*—Automated highway, backstepping, game theory, minimax control, nonlinear observers, protection/safety, road vehicles control.**

I. Introduction

IN MOST of the the automated highway system (AHS) architectures of the California PATH program, traffic is organized into platoons of closely spaced vehicles [1]. The tight spacing between vehicles within a platoon prevents collisions at high relative velocities. The gaps between platoons are large to ensure that a platoon will have time to stop, avoiding a high-speed collision, even if the platoon ahead of it brakes abruptly.

Platoons can perform three basic maneuvers [2], [3]: join, split, and change lane. In a join, two platoons join to form a single platoon; in a split, one platoon breaks into two; and in a change lane, a platoon switches into an adjacent lane. Under normal operations, platoons with more than one vehicle cannot change lanes. Before the change lane maneuver can occur, the platoon must be at a safe distance from the platoons in the adjacent lane. The decelerate to change lane maneuver creates this safe spacing.

The behavior of a platoon is determined by the control law that is applied to its leader. Under normal operation, there are five control laws for the leader of a platoon: leader law, join law, split law, decelerate to change lane law, and change lane law. The leader law is used to keep a platoon traveling at a target velocity and at a safe distance from the platoon ahead. The join, split, and decelerate to change lane laws are used to regulate the three longitudinal maneuvers: join, split, and decelerate to change lane, respectively. The change lane law controls the lateral motion of a vehicle when it goes from one lane to another.

A previous controller design relied on the use of nominal open-loop trajectories that the platoon executing the control law attempted to track [4]. The control laws were safe and comfortable for passengers under normal circumstances. However, since safety was not explicitly considered in the controller desing, if the platoon which is ahead of the one performing the maneuver undergoes large accelerations or decelerations, comfort and safety can be compromised. If the acceleration capabilities of the platoon tracking the trajectory are lower than expected, the maneuvers may not complete at all.

In [5], feedback based controllers for the join, split, and decelerate to change lane laws were proposed. Controllers were robust to such factors as deceleration of the lead platoon and variable acceleration capability. Instead of using timed trajectories for the trail platoon to follow, these controllers use a finite-state machine that switches among feedback laws, in order to keep the velocity of the platoon within a safety limit. The controllers also keep the jerk and acceleration within comfort boundaries, except when safety becomes critical. Completion of the maneuvers in this design does not depend on meeting a desired open-loop acceleration trajectory.

This paper presents a unified control strategy for the single lane control laws: leader, join, split, and decelerate to change lane. The controller design is realized in two stages. In the first stage, for each control law, a desired velocity profile for the platoon leader is derived. This profile guarantees that high-speed collision will be avoided under single-lane disturbances. Whenever safety is not compromised, the platoon will attempt to achieved a target velocity and separation from the platoon ahead in minimum time and by using acceleration and jerk within comfort limits. In the second stage, a nonlinear velocity tracking controller is designed. This controller allows the platoon to track the desired velocity within a given error bound. The control laws prevent even low-speed collisions in all but the most extreme cases of lead platoon deceleration. If the platoon ahead applies and holds maximum braking, a collision could still occur, but the relative velocity at impact will be within a specified acceptable limit. As in [5], the cost of improved safety and comfort is in the increase time that a maneuver takes to be completed.

Simplicity is the main advantage of the new strategy over the controllers presented in [5]. The same controller is used for all the control laws, thus reducing the computational effort in the implementation and simplifying the performance

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and verification analysis. As the platoon safe velocity region is derived using the same approach for all control laws, transitions between control laws are also guaranteed to be safe. This is not the case of the previous designs [5], [4]. In this paper, a rigorous proof that the control laws are safe is also presented. This proof that was not included in [5].

The controller proposed in this paper can be used under normal or degraded conditions by simply changing parameters. Its use to control the longitudinal behavior during change lane maneuvers is also possible.

This paper is organized as follows. Section II presents a theorem that establishes sufficient conditions for a control law to be safe. In Section III, for each control law, the desired velocity profiles for the platoon leader are derived. Section IV describes the velocity profile tracking controller design. Simulation results are presented in Section V. The proof of the theorem of Section II is contained in the Appendix.

II. Safe Control Laws

In this section, we derive conditions to guarantee that platoon control laws are safe. The notion of safety is that the platoon performing the control law will not collide with the platoon ahead at a relative velocity greater than a prescribed limit, v_{allow}. Safe control laws are accomplished under the following assumptions.

1) The acceleration of any vehicle lies in the range $[-a_{\min}, a_{\max}]$.
2) The velocity of any vehicle is always positive, i.e., reverse motions will never occur.
3) The maximum braking acceleration $-a_{\min}$ can be achieved d seconds after a full braking command is issued.

Consider two platoons, the lead platoon and the trail platoon, with the latter being behind the former in the same lane. Let $x_{\text{trail}}(t)$ and $x_{\text{lead}}(t)$ be the positions at time t of the trail and the lead platoons, respectively, and let $\dot{x}_{\text{lead}}(t)$,$\dot{x}_{\text{trail}}(t)$, $\ddot{x}_{\text{lead}}(t)$, and $\ddot{x}_{\text{trail}}(t)$ denote the first and second time derivatives of these positions at time t. $\dot{x}_{\text{lead}}(t)$ and $\dot{x}_{\text{trail}}(t)$ will also be denoted by $v_{\text{lead}}(t)$ and $v_{\text{trail}}(t)$, respectively. Let the accelerations of the lead platoon be $w(t)$ and that of the trail platoon be $u_a(t)$.

The dynamics are given by

$$\ddot{x}_{\text{lead}}(t) = w(t) \tag{1}$$

$$\ddot{x}_{\text{trail}}(t) = u_a(t) \tag{2}$$

where $w(t)$, $u_a(t) \in [-a_{\min}, a_{\max}]$ for all time t, and $w(t)$ and $u_a(t)$ are such that $\dot{x}_{\text{lead}}(t)$ and $\dot{x}_{\text{trail}}(t)$ remain positive for all t.

Define the relative distance between the platoons to be

$$\Delta x(t) := x_{\text{lead}}(t) - x_{\text{trail}}(t). \tag{3}$$

Notice that since the dynamics of the lead and the trail platoons are independent of the absolute positions, x_{lead} (or x_{trail}) the relevant dynamics of the platoons can be described by the dynamics of the relative displacements and of the absolute velocity of the lead platoon. Hence, the dynamics of the relative motion of two platoons is given by

$$\Delta\dot{x}(t) := \dot{x}_{\text{lead}}(t) - \dot{x}_{\text{trail}}(t) \tag{4}$$

$$\Delta\ddot{x}(t) := \ddot{x}_{\text{lead}}(t) - \ddot{x}_{\text{trail}}(t) = w(t) - u_a(t) \tag{5}$$

$$\dot{v}_{\text{lead}} := w(t) \tag{6}$$

where $\Delta\dot{x}(t)$ and $\Delta\ddot{x}(t)$ denote the relative velocity and the relative acceleration between the platoons, $v_{\text{lead}} = \dot{x}_{\text{lead}}$ is the velocity of the lead platoon, and $\dot{v}_{\text{lead}}$ is its time derivative.

Definition 2.1 (Unsafe Impact): An unsafe impact is said to happen at time t if

$$\Delta x(t) \leq 0 \quad \text{and} \quad -\Delta\dot{x}(t) \geq v_{\text{allow}} \tag{7}$$

with $v_{\text{allow}} \geq 0$ being the maximum allowable impact velocity.

We shall use the notation $X_{MS} \subset \Re^3$ to denote the set of all triples $(\Delta x, \Delta\dot{x}, v_{\text{lead}})$ such that (7) is not satisfied and $v_{\text{lead}} \geq 0$.

Definition 2.2 (Safe Control): A control law for the trail platoon is said to be safe for an initial condition $(\Delta x(0), \Delta\dot{x}(0), v_{\text{lead}}(0))$ if the following is true: For any arbitrary lead platoon acceleration $w(\tau); \tau \geq 0$ such that $w(\tau) \in [-a_{\min}, a_{\max}]$ and $v_{\text{lead}}(\tau) \geq 0$, $(\Delta x(t), \Delta\dot{x}(t), v_{\text{lead}}(t)) \in X_{MS}$ for all $t \geq 0$.

The notion of safety is therefore given by the condition that the trail platoon will not collide with the lead platoon at a relative speed greater than the prescribed $v_{\text{allow}} \geq 0$, regardless of the behavior of the lead platoon. The choice of v_{allow} depends on the particular maneuver. v_{allow} determines the tradeoffs between the time the maneuver takes to complete and the risk of injuries. For example, for a join to be completed in a reasonable time while maintaining safety, v_{allow} is set to be a positive number; whereas in a split, v_{allow} is set to be zero since the time a split takes to complete is relatively insensitive to v_{allow}.

The following theorem establishes a subset of X_{MS} such that a control law exists which is safe for any initial conditions $(\Delta x(0), \Delta\dot{x}(0), v_{\text{lead}}(0))$ that lies in this subset.

Theorem 2.1: Let $X_{\text{safe}} \in \Re^3$ be the set of $(\Delta x, \Delta\dot{x}, v_{\text{lead}})$ that satisfy (8) shown at the bottom of the page, where v_{allow} is the maximum relative speed between the lead and trail platoons at which an impact can occur safely, and d is the delay for maximum deceleration to be achieved when a maximum braking command is issued.

There exists a control law that is safe for any initial condition $(\Delta x(0), \Delta\dot{x}(0), v_{\text{lead}}(0)) \in X_{\text{safe}}$, in the sense of Definition 2.2.

Moreover, any control law that applies maximum braking whenever $(\Delta x(t), \Delta\dot{x}(t), v_{\text{lead}}(t)) \notin X_{\text{safe}}$ is safe for any

$$-\Delta\dot{x} < \max\begin{cases} -(a_{\max} + a_{\min})\,d - v_{\text{lead}} + \sqrt{2a_{\min}\Delta x + v_{\text{lead}}^2 + v_{\text{allow}}^2 + a_{\min}(a_{\max} + a_{\min})d^2} \\ -(a_{\max} + a_{\min})\,d + v_{\text{allow}} \end{cases} \tag{8}$$

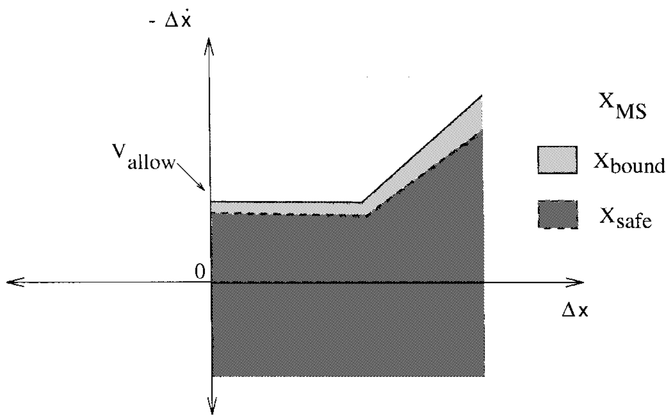

Fig. 1. Relationships between X_{MS}, X_{bound} and X_{safe}. Notice that the vertical axis is "-" relative velocity, i.e., $-\Delta\dot{x}$.

initial condition $(\Delta x(0), \Delta\dot{x}(0), v_{\text{lead}}(0)) \in X_{\text{safe}}$. Under such control law, $(\Delta x(t), \Delta\dot{x}(t), v_{\text{lead}}(t))$ satisfies

$$-\Delta\dot{x}(t) < \max(-v_{\text{lead}}(t) + \sqrt{2a_{\min}\Delta x(t) + v_{\text{lead}}^2(t) + v_{\text{allow}}^2(t)}, v_{\text{allow}}). \tag{9}$$

We shall denote the set of all $(\Delta x, \Delta\dot{x}, v_{\text{lead}})$ that satisfy (9) by X_{bound}, a subset of X_{MS}.

Notice that $X_{\text{safe}} \subset X_{\text{bound}} \subset X_{MS}$. The relations between X_{MS}, X_{safe} and X_{bound} are illustrated in Fig. 1, when v_{lead} is constant.

Proof: See the Appendix.

Remark 2.1:

1) Theorem 2.1 will be used to guarantee that a control law for a maneuver is safe. In the control laws that we propose in this paper, whenever $(\Delta x(t), \Delta\dot{x}(t), v_{\text{lead}}(t)) \notin X_{\text{safe}}$, maximum braking is applied. Hence, by Theorem 2.1, if $(\Delta x(0), \Delta\dot{x}(0), v_{\text{lead}}(0)) \in X_{\text{safe}}$, the safe control laws maintain the relationship, $(\Delta x(\tau), \Delta\dot{x}(\tau), v_{\text{lead}}(\tau)) \in X_{\text{bound}} \subset X_{MS}$ for all $\tau \geq 0$. Thus, an unsafe impact will not occur.
2) Theorem 2.1 can also be used by the maneuver planning supervisor to determine if a maneuver should be allowed to take place from the safety point of view. This is accomplished by checking if the initial condition lies in the set X_{safe} for the given v_{allow}.
3) Notice that when the delay $d = 0$, i.e., maximum braking can be achieved instantaneously, the sets X_{safe} and X_{bound} are the same. Thus, when $d = 0$, $\overline{X_{\text{bound}}}$, the closure of X_{bound} is invariant if the control law consists of applying maximum braking whenever $(\Delta x, \Delta\dot{x}, v_{\text{lead}})$ lies outside X_{bound}. Since $\overline{X_{\text{bound}}} \subset X_{MS}$, an unsafe impact will not occur. However, since maximum braking cannot be achieved until after a delay of d seconds, the condition to apply maximum braking is more stringent (outside X_{safe}). Indeed, the relationship between the boundaries ∂X_{safe} and $\partial X_{\text{bound}}$ of X_{safe} and X_{bound}, respectively, is such that if maximum braking is applied at time t when $(\Delta x(t), \Delta\dot{x}(t), v_{\text{lead}}(t)) \in \partial X_{\text{safe}}$, and for $\tau \in [0, d]$, the worst case scenario which is $w(\tau) = -a_{\min}$ and $u_a(\tau) = a_{\max}$ takes place, then $(\Delta x(t+d), \Delta\dot{x}(t+d), v_{\text{lead}}(t+d)) \in \partial X_{\text{bound}}$.

III. Velocity Profiles

In this section we express the relative motion of platoons as desired trajectories profiles in the state space $(\Delta x(t), \Delta\dot{x}(t), v_{\text{lead}}(t))$ that satisfy both requirements: safety and time-optimality. We will assume that, whenever safety is not compromised, platoons should keep the acceleration and jerk within comfort bounds.

A. Join Law

In a join maneuver, the goal of its control law is to decrease the initial relative displacement between the lead platoon and the trail platoon $\Delta x(0)$ to the desired intraplatoon spacing Δx_{join}. The relative velocity $\Delta\dot{x}(t)$ should be zero at the end of the join maneuver. According to Theorem 2.1, the resulting trajectory of $(\Delta x(t), \Delta\dot{x}(t), v_{\text{lead}}(t))$ must be within the safety set X_{bound}.

In order to decrease the time the join maneuver takes to complete, the relative velocity between the trail and lead platoons, $-\Delta\dot{x}$, should be maximized while observing the safety limits. This suggest that the state $(\Delta x(t), \Delta\dot{x}(t), v_{\text{lead}}(t))$ of the join maneuver should be kept, as much as possible, in the boundary ∂X_{safe} of the safety set X_{safe} in Theorem 2.1. This boundary consists of two smooth portions:

1) In the first portion, the trail platoon is far enough from the lead platoon so that maximum deceleration will stop the lead platoon before the trail platoon hits it at v_{allow}, if a collision occurs.
2) The other portion of the maximum safe velocity curve represents the case when full braking does not stop the lead platoon before the trail platoon hits it at v_{allow}, if a collision occurs.

Therefore, according with (8), the maximum safe velocity curve v_{safe} of the trail platoon for a given v_{lead} and Δx is shown in (10) at the bottom of the page.

To finish the join maneuver in minimum time, it is necessary to slow the trail platoon to v_{lead} at the end of the join. It is imposed that the trail platoon should decelerate at the maximum comfortable level. The velocity in the deceleration

$$v_{\text{safe}}(v_{\text{lead}}, \Delta x) = \max \begin{cases} -(a_{\max} + a_{\min})\, d + \sqrt{2a_{\min}\Delta x + v_{\text{lead}}^2 + v_{\text{allow}}^2 + a_{\min}(a_{\max} + a_{\min})\, d^2} \\ -(a_{\min} + a_{\max})\, d + v_{\text{lead}} + v_{\text{allow}}. \end{cases} \tag{10}$$

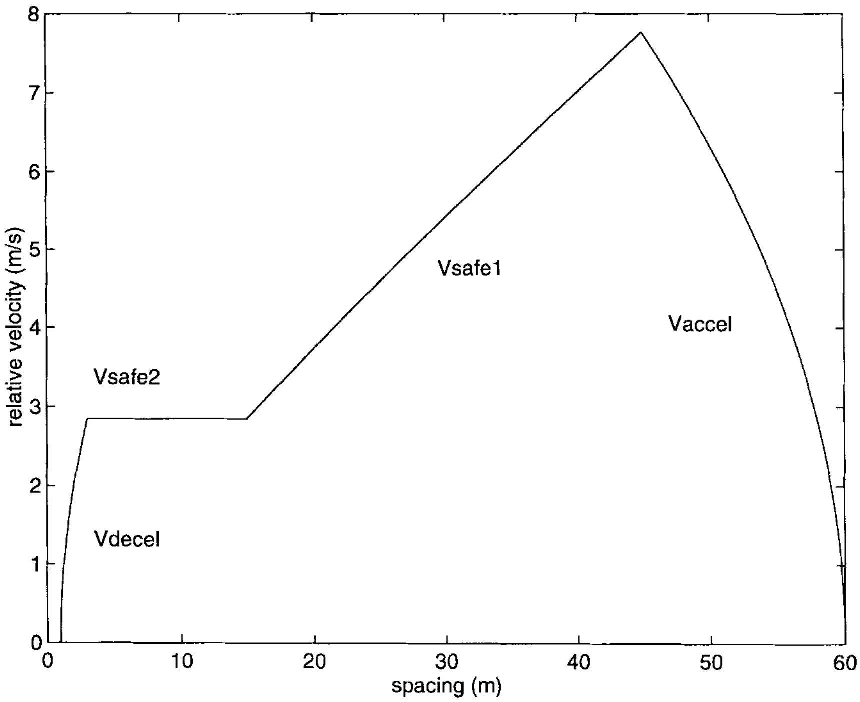

Fig. 2. Basic velocity profile for 60-m initial spacing. The lead platoon is moving at a constant velocity of 25 m/s.

curve, $v_{\min}$, written as a function of v_{lead} and Δx is

$$v_{\min}(v_{\text{lead}}, \Delta x) = \min \begin{cases} v_{\text{lead}} + \sqrt{2a_{\text{com}}(\Delta x - \Delta x_{\text{join}})} \\ v_{\text{fast}}. \end{cases} \tag{11}$$

where a_{com} is the magnitude of the comfort acceleration and deceleration, Δx_{join} the desired intraplatoon distance and v_{fast} is the maximum recommend velocity for a platoon to travel on the highway.

In order for the join control law to be safe and to allow the maneuver to be completed in minimum time, the velocity of the trail platoon should satisfy

$$v_d(v_{\text{lead}}, \Delta x) = \min(v_{\min}, v_{\text{safe}}).$$

Equations (10) and (11) define a desired velocity profile for the trail platoon during a safe join law. Fig. 2 shows an example of this desired velocity profile in the $\Delta x(\cdot)$ versus $\Delta \dot{x}(\cdot)$ phase plane. For the profile in Fig. 2 it is assumed that the lead platoon is traveling at constant velocity. The acceleration portion will be produced by the velocity tracking controller to be described in the next section.

The desired phase-plane trajectory for the trail platoon velocity includes abrupt changes in acceleration at the points where sections of the curve intersect. It is convenient to smooth these transitions so as not to violate jerk comfort constraints. Cubic splines are used for this purpose [6].

B. Split Law

In the split maneuver the goal is to increase the distance between the lead and trail platoon, Δx, to a desired value Δx_{split}. Platoons' relative speed, $\Delta \dot{x}$, must necessarily be increased to accomplish this increment. For this reason, in most cases, the velocity of the trail platoon will be lower than the velocity of the lead platoon, and thus the threat of high-speed collisions will be inherently reduced.

A similar approach to the one used in the join law can also be used for the split law. Two boundary curves are established for the velocity of the trail platoon, v_{trail}. The first one, related to safety, is derived from (8) by assuming $v_{\text{allow}} = 0$, where v_{allow} is the allowable impact relative velocity. Thus, for a given v_{lead} and Δx, the maximum velocity of the trail platoon for the split law to be safe is

$$\begin{aligned} v_{\text{safe}}&(v_{\text{lead}}, \Delta x) \\ &= -(a_{\min} + a_{\max})\, d \\ &\quad + \sqrt{2a_{\min}\Delta x + v_{\text{lead}}^2 + a_{\min}(a_{\max} + a_{\min})\, d^2}. \end{aligned} \tag{12}$$

The other boundary curve is related to time-optimality. This curve establishes a lower bound on the velocity of the trail platoon. To determine this lower bound, it is assumed that, for a given v_{lead} and Δx, if the trail platoon is traveling at this minimum velocity, then it will reach the desired intraplatoon distance Δx_{split} with null relative velocity by applying maximum comfort acceleration. It is also assumed that there exists a minimum velocity v_{slow} below which it is not recommended to travel on the highway under normal circumstances. The minimum velocity of the trail platoon is therefore given by

$$v_{\min}(v_{\text{lead}}, \Delta x) = \max \begin{cases} v_{\text{lead}} - \sqrt{2a_{\text{com}}(\Delta x_{\text{split}} - \Delta x)} \\ v_{\text{slow}}. \end{cases} \tag{13}$$

At any particular state $(\Delta x, \Delta \dot{x}, v_{\text{lead}})$ of a split maneuver, the velocity of the trail platoon should satisfy the safety requirements, therefore from (12) and (13)

$$v_d(v_{\text{lead}}, \Delta x) = \min(v_{\min}, v_{\text{safe}}).$$

C. Decelerate to Change Lane Law

The decelerate to change lane control law attempts to create a safe distance between platoons in different lanes before any actual change lane maneuver can take place. The decelerate to change lane law can be treated similarly to the split law. The only distinction in terms of safety is that the maximum velocity for the trail platoon has to be calculated for two lead platoons, the one that is in the same lane as the platoon attempting to change lane and the one that is in the adjacent lane. The maximum safe velocity for the trail platoon is therefore

$$v_{\text{safe}}(v_{\text{lead}}, \Delta x, v_{\text{next}}, \Delta x_{\text{next}}) = \min\begin{cases} -(a_{\min}+a_{\max})\,d \\ \quad +\sqrt{2a_{\min}\Delta x+v_{\text{lead}}^2+a_{\min}(a_{\max}+a_{\min})\,d^2} \\ -(a_{\min}+a_{\max})\,d \\ \quad +\sqrt{2a_{\min}\Delta x_{\text{next}}+v_{\text{next}}^2+a_{\min}(a_{\max}+a_{\min})\,d^2} \end{cases} \tag{14}$$

where v_{next} is the velocity of the lead platoon in the adjacent lane and Δx_{next} is the longitudinal spacing between the platoon in the trail platoon and the lead platoon in the adjacent lane.

The minimum velocity of the trail platoon is established in the same way as in the previous control law, but considering the target velocity and distance with respect to the platoon in the adjacent lane. Thus

$$v_{\min}(v_{\text{next}}, \Delta x_{\text{next}}) = \max\begin{cases} v_{\text{next}} - \sqrt{2a_{\text{com}}(\Delta x_{\text{change}} - \Delta x_{\text{next}})} \\ v_{\text{slow}}. \end{cases} \tag{15}$$

At any particular stage of a decelerate to change lane maneuver, the velocity of the trail platoon should satisfy the safety requirements, therefore from (14) and (15)

$$v_d(v_{\text{lead}}, \Delta x, v_{\text{next}}, \Delta x_{\text{next}}) = \min(v_{\min}, v_{\text{safe}}).$$

D. Leader Law

The leader law is intended to keep a platoon traveling on a highway at a target velocity and at a safe distance from the platoon ahead. As transitions from other control laws to the leader law can happen at any point in the safe state set X_{safe}, it is also necessary to guarantee the safety of the leader law. The safety of a platoon executing the leader law can be analyzed in a similar way to a platoon that is involved in a join maneuver. The target velocity for a platoon leader executing the leader law is no longer the velocity of the platoon ahead, but some desired velocity v_{link} given by a highway traffic controller [7]. Under normal conditions the platoon leader should keep at least a distance Δx_{leader} from the platoon ahead.

The maximum safe velocity curve v_{safe} for a platoon in leader law, given v_{lead} and Δx, is (16) shown at the bottom of the page.

The target velocity for the a platoon under the leader control law is given by

$$v_{\min}(v_{\text{lead}}, \Delta x) = \min\begin{cases} v_{\text{lead}} - \sqrt{2a_{\text{com}}(\Delta x_{\text{leader}} - \Delta x)}; \\ \qquad \text{if } \Delta x_{\text{leader}} \geq \Delta x \\ v_{\text{link}}; \quad \text{else.} \end{cases} \tag{17}$$

It should be noticed that this term is designed to allow the platoon to travel at the link layer target speed v_{link} only when its separation from the platoon ahead is larger than the desired one, i.e., $\Delta x > \Delta x_{\text{leader}}$.

The desired velocity for a platoon under the leader law is therefore

$$v_d(v_{\text{lead}}, \Delta x) = \min(v_{\min}, v_{\text{safe}}).$$

It is also important to remark that no matter which control law is being executed, whenever the relative distance between platoons Δx is larger than the detection range of the relative position sensor, a transition to the leader law should be taken. This provision should be considered in the supervisor of the control laws that will be described subsequently.

IV. Velocity Profile Tracking Control

In the previous section we established velocity profiles for a platoon in order for a control law to be safe and fast. In this section we introduce a velocity tracking control law that commands the actual velocity of the trail platoon to follow these velocity profiles.

We assume that the positions and velocities of both the lead and the trail platoons are measured quantities as is the acceleration of the trail platoon which is executing any of the control laws. An estimate of the acceleration of the lead platoon is also necessary in the control law, since the velocity profile of a maneuver is a function of the lead platoon velocity. The proposed control law combines an observer for the lead platoon state and a nonlinear controller design. This is accomplished via the backstepping procedure and the use of tuning functions. The jerk of the lead platoon depends on the specific maneuver that it is undergoing and so it is modeled as noise.

A. Backstepping Design

Let $v_d(\Delta x, v_{\text{lead}})$ be the value of the desired velocity flow field for the trail platoon when the displacement between the

$$v_{\text{safe}}(v_{\text{lead}}, \Delta x) = \max\begin{cases} -(a_{\max}+a_{\min})\,d \\ \quad +\sqrt{2a_{\min}\Delta x + v_{\text{lead}}^2 + v_{\text{allow}}^2 + a_{\min}(a_{\max}+a_{\min})\,d^2} \\ -(a_{\min}+a_{\max})\,d + v_{\text{lead}} + v_{\text{allow}}. \end{cases} \tag{16}$$

lead and trail platoons is

$$\Delta x = x_{\text{lead}} - x_{\text{trail}}$$

and the lead platoon velocity is v_{lead}. Define the velocity error by

$$e := \dot{x}_{\text{trail}} - v_d(\Delta x, v_{\text{lead}}).$$

The velocity error dynamics is given by

$$\dot{e} = \ddot{x}_{\text{trail}} - \begin{pmatrix} \frac{\partial v_d}{\partial \Delta x} & \frac{\partial v_d}{\partial v_{\text{lead}}} \end{pmatrix} \begin{pmatrix} v_{\text{lead}} - \dot{x}_{\text{trail}} \\ \dot{v}_{\text{lead}} \end{pmatrix}.$$

To apply the backstepping procedure, suppose first that the trail platoon acceleration $\ddot{x}_{\text{trail}}$ can be controlled directly. In this case an appropriate control for $\ddot{x}_{\text{trail}}$ is

$$\Gamma(\Delta x, v_{\text{lead}}, \dot{x}_{\text{trail}}, \hat{a}_{\text{lead}}) := -\lambda_1 e + \begin{pmatrix} \frac{\partial v_d}{\partial \Delta x} & \frac{\partial v_d}{\partial v_{\text{lead}}} \end{pmatrix} \begin{pmatrix} v_{\text{lead}} - \dot{x}_{\text{trail}} \\ \hat{a}_{\text{lead}} \end{pmatrix} \tag{18}$$

where $\hat{a}_{\text{lead}}$ is an estimate of the acceleration of the lead platoon.

Define now $\tilde{a}_{\text{lead}} := a_{\text{lead}} - \hat{a}_{\text{lead}}$ to be the lead platoon acceleration estimation error, and

$$\tilde{\Gamma}(t) := \ddot{x}_{\text{trail}}(t) - \Gamma(\Delta x(t), v_{\text{lead}}(t), \dot{x}_{\text{trail}}(t), \hat{a}_{\text{lead}}(t))$$

to be the discrepancy between the actual trail platoon acceleration and the ideal one. Then, the velocity error dynamics is given by

$$\dot{e} = -\lambda_1 e - \frac{\partial v_d}{\partial v_{\text{lead}}} \tilde{a}_{\text{lead}} + \tilde{\Gamma}.$$

Consider now the dynamics of $\tilde{\Gamma}$

$$\dot{\tilde{\Gamma}} = j_{\text{trail}} - \begin{pmatrix} \frac{\partial \Gamma}{\partial \Delta x} & \frac{\partial \Gamma}{\partial v_{\text{lead}}} & \frac{\partial \Gamma}{\partial \dot{x}_{\text{trail}}} & \frac{\partial \Gamma}{\partial \hat{a}_{\text{lead}}} \end{pmatrix} \cdot \begin{pmatrix} v_{\text{lead}} - \dot{x}_{\text{trail}} \\ a_{\text{lead}} \\ \ddot{x}_{\text{trail}} \\ \dot{\hat{a}}_{\text{lead}} \end{pmatrix}$$

where $j_{\text{trail}} := d^3 x_{\text{trail}}/dt^3$ is the control jerk of the trail platoon and $\dot{\hat{a}}_{\text{lead}}$ is the time derivative of the estimate of the lead platoon's acceleration. The expression for the latter depends on the implementation of the lead platoon state observer and will be defined later when the observer is presented.

We propose the following control for j_{trail}:

$$j_{\text{trail}} = -\lambda_2 \tilde{\Gamma} - \beta e + \begin{pmatrix} \frac{\partial \Gamma}{\partial \Delta x} & \frac{\partial \Gamma}{v_{\text{lead}}} & \frac{\partial \Gamma}{\partial \dot{x}_{\text{trail}}} & \frac{\partial \Gamma}{\partial \hat{a}_{\text{lead}}} \end{pmatrix} \cdot \begin{pmatrix} v_{\text{lead}} - \dot{x}_{\text{trail}} \\ \hat{a}_{\text{lead}} \\ \ddot{x}_{\text{trail}} \\ \dot{\hat{a}}_{\text{lead}}^{\sim} \end{pmatrix} \tag{19}$$

where $\dot{\hat{a}}_{\text{lead}}^{\sim}$ is an estimate of the time derivative of the lead platoon acceleration. When $\hat{a}_{\text{lead}}$ is estimated using a full-order observer, $\dot{\hat{a}}_{\text{lead}}^{\sim} = \dot{\hat{a}}_{\text{lead}}$; when $\hat{a}_{\text{lead}}$ is estimated using a reduced-order observer, their difference is proportional to the error in the estimate of the lead platoon acceleration. Thus

$$\dot{\hat{a}}_{\text{lead}} - \dot{\hat{a}}_{\text{lead}}^{\sim} = d_1 \tilde{a}_{\text{lead}} \tag{20}$$

where $d_1 = 0$ when a full-order observer is used and is a known constant when a reduced-order observer is used.

The dynamics of $\tilde{\Gamma}$ under (19) becomes

$$\dot{\tilde{\Gamma}} = -\beta e - \lambda_2 \tilde{\Gamma} - \left(\frac{\partial \Gamma}{\partial v_{\text{lead}}} + \frac{\partial \Gamma}{\partial \hat{a}_{\text{lead}}} d_1 \right) \tilde{a}_{\text{lead}}.$$

Using the notation in the equation shown at the bottom of the page, the combined dynamics of e and $\tilde{\Gamma}$ are given by

$$\frac{d}{dt} \begin{pmatrix} e \\ \tilde{\Gamma} \end{pmatrix} = \begin{pmatrix} -\lambda_1 & 1 \\ -\beta & -\lambda_2 \end{pmatrix} \begin{pmatrix} e \\ \tilde{\Gamma} \end{pmatrix} + \boldsymbol{g} \tilde{a}_{\text{lead}}. \tag{21}$$

Notice that the state evolution matrix in (21) is stable when λ_1, λ_2 and β are positive. The design values of these parameters can be obtained by minimizing the effect of $\tilde{a}_{\text{lead}}$ on e using linear methods and assuming constant values of $\boldsymbol{g}(t)$. In particular, let $[\overline{g}_1, \overline{g}_2]^T$ be some average design constants of the vector $\boldsymbol{g}$, the transfer function from $\tilde{a}_{\text{lead}}$ to e is

$$\frac{E(s)}{\tilde{A}_{\text{lead}}(s)} = \overline{g}_1 \frac{s + \overline{g}_2/\overline{g}_1 + \lambda_2}{s^2 + (\lambda_1 + \lambda_2)s + \lambda_1\lambda_2 + \beta}. \tag{22}$$

Thus λ_2 has the effect of shifting the zero to the left, and each of the constants λ_1, λ_2, and β can also be used to move the poles of this transfer function to the left. The low-frequency gain is

$$\frac{\overline{g}_1 \lambda_2 + \overline{g}_2}{\lambda_1 \lambda_2 + \beta}.$$

$$\boldsymbol{g} := \begin{pmatrix} -\frac{\partial v_d}{\partial v_{\text{lead}}} \\ -\frac{\partial \Gamma}{\partial v_{\text{lead}}} - d_1 \frac{\partial \Gamma}{\partial \hat{a}_{\text{lead}}} \end{pmatrix} = \begin{pmatrix} -\frac{\partial v_d}{\partial v_{\text{lead}}} \\ -(\lambda_1 + d_1) \frac{\partial v_d}{\partial v_{\text{lead}}} - \frac{\partial v_d}{\partial \Delta x} - \frac{\partial^2 v_d}{\partial \Delta x \partial v_{\text{lead}}} (v_{\text{lead}} - \dot{x}_{\text{trail}}) - \frac{\partial^2 v_d}{\partial v_{\text{lead}}^2} \hat{a}_{\text{lead}} \end{pmatrix}$$

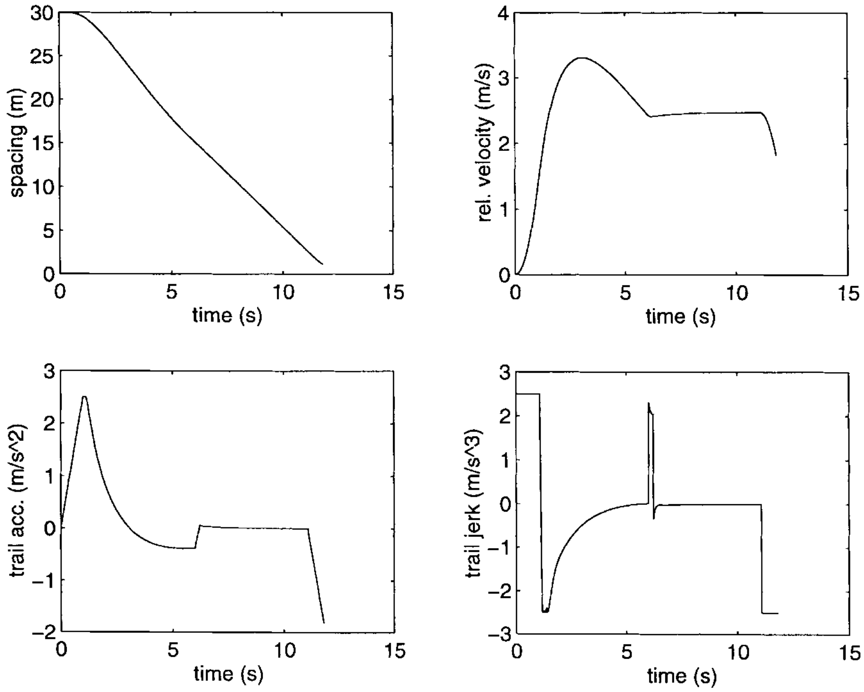

Fig. 3. Simulation results of join from 30-m initial spacing: The initial velocity of both lead and trail platoons was 25 m/s. In the graphs, spacing refers to Δx, and relative velocity is $v_{\text{trail}} - v_{\text{lead}}$.

B. Lead Platoon State Observers

The velocity profile tracking controller makes use of the estimate of the acceleration of the lead platoon, which is not measured. We now present observers that estimate it. A full-order observer is presented first, followed by the reduced-order observer.

The lead platoon dynamics is given by

$$\frac{d^3}{dt^3}x_{\text{lead}} = j_{\text{lead}}(t) \tag{23}$$

where j_{lead} is the jerk input to the lead platoon. Let

$$A := \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad B := \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad C := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

1) Full-Order Observer: A full-order state observer for the lead platoon acceleration is

$$\begin{aligned} \frac{d}{dt}\hat{\boldsymbol{x}}_{\text{lead}} &= A\hat{\boldsymbol{x}}_{\text{lead}} - L\left(\begin{pmatrix} x_{\text{lead}} \\ v_{\text{lead}} \end{pmatrix} - C\hat{\boldsymbol{x}}_{\text{lead}}\right) + \boldsymbol{q}(t) \\ \hat{a}_{\text{lead}} &:= (0 \quad 0 \quad 1)\hat{\boldsymbol{x}}_{\text{lead}} \end{aligned} \tag{24}$$

where $\hat{\boldsymbol{x}}_{\text{lead}} \in \mathcal{R}^3$ is the state estimate, the observer gain $L \in \mathcal{R}^{3\times 2}$ is such that $A - LC$ is asymptotically stable, and $\boldsymbol{q}(t)$ is a tuning function to be determined. For the full-order observer, $\dot{\hat{a}}_{\text{lead}}$ can be determined without error using known quantities from (24), i.e., $d_1 = 0$ in (20).

The dynamics of the acceleration estimation error $\tilde{a}_{\text{lead}}$ is given by

$$\begin{aligned} \dot{\tilde{\boldsymbol{x}}}_{\text{lead}} &= (A - LC)\tilde{\boldsymbol{x}}_{\text{lead}} + Bj_{\text{lead}}(t) - \boldsymbol{q}(t) \\ \tilde{a}_{\text{lead}} &= (0 \quad 0 \quad 1)\tilde{\boldsymbol{x}}_{\text{lead}} \end{aligned} \tag{25}$$

where $\tilde{\boldsymbol{x}}_{\text{lead}} := (x_{\text{lead}}\ v_{\text{lead}}\ a_{\text{lead}})^T - \hat{\boldsymbol{x}}_{\text{lead}}$.

2) Reduced-Order Observer: The full-order observer estimates the position and velocity of the lead platoon (which are measured quantities) in addition to the unmeasured acceleration a_{lead}. A reduced-order observer which estimates only the acceleration can be similarly designed as follows:

$$\begin{aligned} \dot{r} &= -L_2 r - (L_1 L_2 \quad L_2^2 + L_1)\begin{pmatrix} x_{\text{lead}} \\ v_{\text{lead}} \end{pmatrix} + q(t) \\ \hat{a}_{\text{lead}} &:= r + L_1 x_{\text{lead}} + L_2 v_{\text{lead}} \end{aligned} \tag{26}$$

where, for this case, L_1 and L_2 are the two components of the matrix L with $L_2 > 0$ for the observer to be stable and $q(t) \in \mathcal{R}$ is a tuning function to be determined.

It can be shown that the acceleration estimation error $\tilde{a}_{\text{lead}} := a_{\text{lead}} - \hat{a}_{\text{lead}}$ is given by

$$\dot{\tilde{a}}_{\text{lead}} = -L_2\tilde{a}_{\text{lead}} + j_{\text{lead}} - q(t). \tag{27}$$

Because of the structure in (26), the reduced-order observer does not allow $\dot{\hat{a}}_{\text{lead}}$ to be computed using known quantities. $\dot{\hat{a}}_{\text{lead}}$ can be estimated by

$$\tilde{\dot{\hat{a}}}_{\text{lead}} := \dot{r} + L_1 v_{\text{lead}} + L_2\hat{a}_{\text{lead}}.$$

Thus $\dot{\hat{a}}_{\text{lead}} - \tilde{\dot{\hat{a}}}_{\text{lead}} = L_2\tilde{a}_{\text{lead}}$, i.e., $d_1 = L_2$ in (20).

C. Stability Analysis

1) Reduced-Order observer: Suppose that the reduced-order observer is used. Consider the Lyapunov function[1]

$$V(e, \tilde{\Gamma}, \tilde{a}_{\text{lead}}) := \tfrac{1}{2}\beta e^2 + \tfrac{1}{2}\tilde{\Gamma}^2 + \tfrac{1}{2}\gamma\tilde{a}_{\text{lead}}^2. \tag{28}$$

[1]For the sake of notation simplicity, with the exception of q, we do not write the time dependence of the variables.

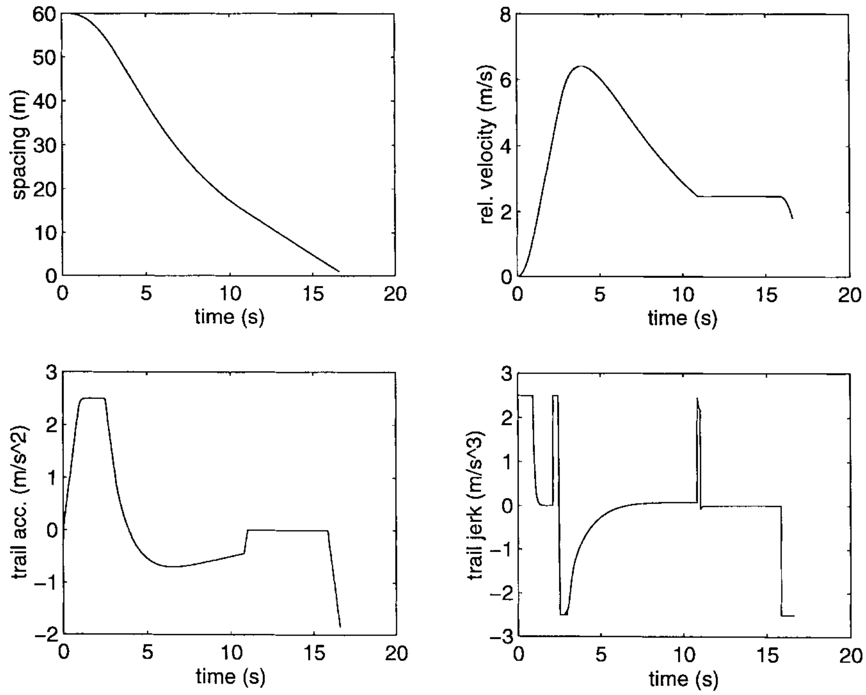

Fig. 4. Simulation results of join from 60-m initial spacing. The initial velocity of both lead and trail platoons was 25 m/s.

Using the dynamics of e, $\tilde{\Gamma}$, and $\tilde{a}_{\text{lead}}$ in (21) and (27)

$$\frac{d}{dt}V(e,\tilde{\Gamma},\tilde{a}_{\text{lead}}) = -\beta\lambda_1 e^2 - \lambda_2\tilde{\Gamma}^2 + (\beta e \quad \tilde{\Gamma})\boldsymbol{g}\tilde{a}_{\text{lead}} - \gamma\tilde{a}_{\text{lead}}q(t) + \gamma j_{\text{lead}}\tilde{a}_{\text{lead}} - \gamma L_2\tilde{a}_{\text{lead}}^2.$$

Thus, if we set the tuning function $q(t)$ in (26) to be

$$q(t) := \frac{1}{\gamma}(\beta e \quad \tilde{\Gamma})\boldsymbol{g}$$

then

$$\frac{d}{dt}V(e,\tilde{\Gamma},\tilde{a}_{\text{lead}}) = -\beta\lambda_1 e^2 - \lambda_2\tilde{\Gamma}^2 - \gamma L_2\tilde{a}_{\text{lead}}^2 + \gamma\tilde{a}_{\text{lead}}j_{\text{lead}}(t).$$

This shows that if $\|j_{\text{lead}}(\cdot)\|_\infty \le j_{\max}$, then

$$\dot{V} \le -2\zeta V + \sqrt{2\gamma}V^{1/2}j_{\max}$$

where $\zeta = \min(\lambda_1,\lambda_2,L_2)$. Similarly

$$\frac{d}{dt}(V^{1/2}) \le -\zeta V^{1/2} + \sqrt{\frac{\gamma}{2}}j_{\max}.$$

Hence, for any initial conditions $(e(0),\tilde{\Gamma}(0),\tilde{a}_{\text{lead}}(0))$, and for any $\epsilon > 0$, there is a time T_1 s.t. if $t \ge T_1$

$$\sqrt{\frac{\beta}{2}}|e(t)| \le V^{1/2}(t) \le \frac{j_{\max}}{\zeta}\sqrt{\frac{\gamma}{2}}(1+\epsilon)$$

or $|e(t)| \le (j_{\max}/\zeta)\sqrt{\gamma/\beta}(1+\epsilon)$ after a long enough time.

2) Full-order Observer: The above analysis is valid for a reduced-order observer and a set of gains, λ_1, λ_2, and β which are positive. It can be generalized to the case when the matrix $\begin{pmatrix}-\lambda_1 & 1\\ -\beta & -\lambda_2\end{pmatrix}$ is stable and a full order observer is used.

Utilizing the fact that a stable linear system admits a positive definite quadratic Lyapunov function, define $F = \begin{pmatrix}-\lambda_1 & 1\\ -\beta & -\lambda_2\end{pmatrix}$, and $A_F = A - LC$ the evolution matrix in the full-order observer in (25). Let $Q \in \mathcal{R}^{2\times 2}$, and $P \in \mathcal{R}^{3\times 3}$ be positive definite symmetric matrices that satisfy the Lyapunov equations

$$QF + F^TQ = -2C_1; \quad PA_F + A_F^TP = -2C_2$$

where $C_1 \in \mathcal{R}^{2\times 2}$ and $C_2 \in \mathcal{R}^{3\times 3}$ are positive definite matrices.

Let $\boldsymbol{s} = (e \quad \tilde{\Gamma})^{\boldsymbol{T}}$. Consider instead of (28), the Lyapunov function

$$V(e,\tilde{\Gamma},\tilde{x}_{\text{lead}}) = \tfrac{1}{2}\boldsymbol{s}^{\boldsymbol{T}}Q\boldsymbol{s} + \gamma\tfrac{1}{2}\tilde{\boldsymbol{x}}_{\text{lead}}^{\boldsymbol{T}}P\tilde{\boldsymbol{x}}_{\text{lead}}. \tag{29}$$

Then, the time derivative of its value is

$$\dot{V} = -\boldsymbol{s}^{\boldsymbol{T}}C_1\boldsymbol{s} - \gamma\tilde{\boldsymbol{x}}_{\text{lead}}^{\boldsymbol{T}}C_2\tilde{\boldsymbol{x}}_{\text{lead}} + \boldsymbol{s}^{\boldsymbol{T}}Q\boldsymbol{g}(0 \quad 0 \quad 1)\tilde{\boldsymbol{x}}_{\text{lead}} - \gamma\tilde{\boldsymbol{x}}_{\text{lead}}^{\boldsymbol{T}}P\boldsymbol{q}(\boldsymbol{t}) + \gamma\boldsymbol{j}_{\text{lead}}(\boldsymbol{t})\boldsymbol{B}^{\boldsymbol{T}}P\tilde{\boldsymbol{x}}_{\text{lead}}.$$

Thus, an appropriate tuning function $\boldsymbol{q}(\boldsymbol{t})$ is

$$\boldsymbol{q}(\boldsymbol{t}) = \frac{(\boldsymbol{g}^{\boldsymbol{T}}Q\boldsymbol{s})}{\gamma}P^{-1}\begin{pmatrix}0\\0\\1\end{pmatrix}. \tag{30}$$

Then

$$\dot{V} = -\boldsymbol{s}^{\boldsymbol{T}}C_1\boldsymbol{s} - \gamma\tilde{\boldsymbol{x}}_{\text{lead}}^{\boldsymbol{T}}C_2\tilde{\boldsymbol{x}}_{\text{lead}} + \gamma\boldsymbol{j}_{\text{lead}}(\boldsymbol{t})\boldsymbol{B}^{\boldsymbol{T}}P\tilde{\boldsymbol{x}}_{\text{lead}}.$$

By choosing $-2C_1 := T_1^{-T}(\Lambda_1 + \Lambda_1^T)T_1^{-1}$, and $-2C_2 := T_2^{-T}(\Lambda_2 + \Lambda_2^T)T_2^{-1}$ where $F = T_1\Lambda_1T_1^{-1}$ and $A_F =$

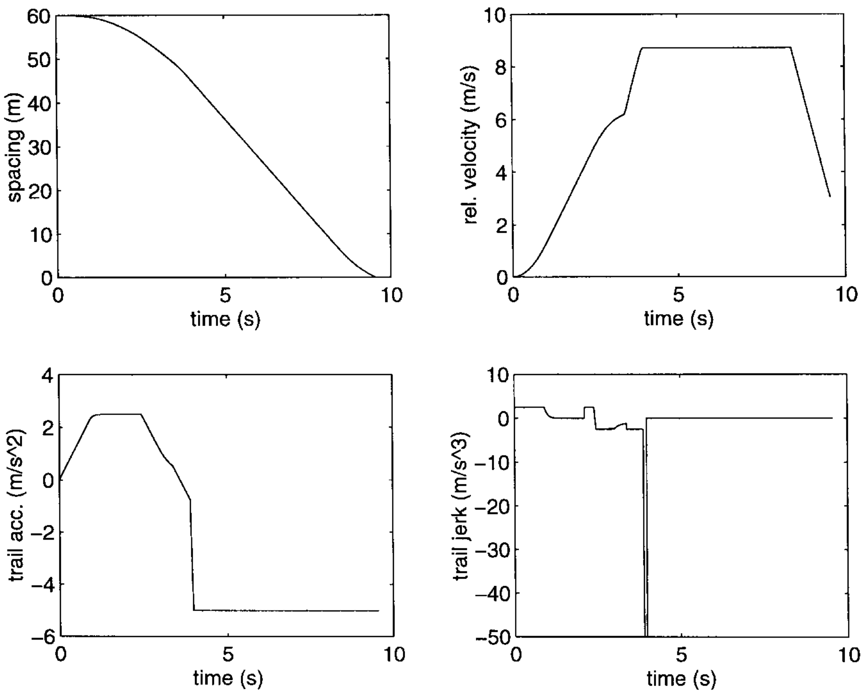

Fig. 5. Simulation results of join from 60-m initial spacing. The initial velocity of both lead and trail platoons was 25 m/s. The lead platoon applied maximum braking at 3.5 s.

$T_2\Lambda_2T_2^{-1}$ are the real decomposition[2] of F and A_F, and T_i^T denotes the transpose of T_i, it can be shown that $Q = T_1^{-T}T_1^{-1}$ and $P = T_2^{-T}T_2^{-1}$. In this case,

$$\dot{V} \le -\zeta V + \tilde{\boldsymbol{x}}_{\text{lead}}^T P_{\cdot 3} j_{\text{lead}}$$

where ζ is the minimum real part of the eigenvalues of F and A_F and $P_{\cdot 3}$ is the third column of P. For any value of V, it can also be shown that

$$e^2 \le \rho V, \quad \text{where} \quad \rho := \frac{2Q_{22}}{Q_{11}Q_{22} - Q_{12}^2}$$

and

$$(P_{\cdot 3}^T \tilde{\boldsymbol{x}}_{\text{lead}})^2 \le \frac{\delta}{\gamma} V, \quad \text{where} \quad \delta := 2P_{\cdot 3}^T P^{-1} P_{\cdot 3}$$

and Q_{ij} are the (i,j)th element of the matrix Q. Thus, an analysis similar to the previous simplified case shows that, for any initial conditions $(\boldsymbol{s}(\mathbf{0}), \tilde{\boldsymbol{x}}_{\text{lead}}(\mathbf{0}))$, and for any $\epsilon > 0$, there is a time T_1 s.t. if $t \ge T_1$

$$\sqrt{\frac{1}{\rho}}\,|e(t)| \le V^{1/2}(t) \le \frac{j_{\max}}{\zeta}\sqrt{\frac{\delta}{\gamma}}\,(1+\epsilon).$$

Therefore $|e(t)| \le (j_{\max}/\zeta)\sqrt{\delta\rho/\gamma}(1+\epsilon)$ after a long enough time.

Issues related to the implementation of the velocity tracking controller can be examined with more detail in [6].

[2] By real decomposition, $F = T_1\Lambda_1T_1^{-1}$, it is meant that T_1 is real and the diagonal of Λ_1 contains the real parts of the eigenvalues of F and the off diagonal part is skew and contains the imaginary parts of the eigenvalues, if any.

V. Simulation Results

The control laws simulation results shown here are from a Matlab program that simulates just two platoons in a maneuver. The program was written to test the control laws for different behaviors of the platoon ahead. All control laws are being implemented in SmartPath [8].

Parameter values for the simulations were set as follows.

1) $a_{\text{com}} = \pm 2$ m/s^2. This is the value used in the current join [4]. It is commonly accepted in the literature. See [9] and [10].
2) $a_{\min} = 5$ m/s^2. This is the absolute value of the maximum deceleration. This value is used in the current join.
3) $a_{\max} = 2.5$ m/s^2. This is a rough approximation based on data presented in [11]. The road is assumed to be flat. The vehicles are assumed to have automatic transmissions in third gear.
4) $j_{\text{com}} = \pm 2.5$ m/s^3. Lygeros and Godbole [4] set the comfortable jerk limit at 5 m/s^3 in the current join. Most examples in the literature suggest the limit is between 2 and 2.5 m/s^3. See [9], [10], and [12].
5) $j_{\max} = \pm 50$ m/s^3. This value was selected as a physical limit on jerk. It is less than the one given in [13].
6) $v_{\text{allow}} = 3$ m/s. The severity of injuries in automobile accidents is measured on the abbreviated injury scale (AIS). Injuries rated from 3 to 6 on this scale are considered serious. Injuries of AIS $= 2$ are moderate. They are not life threatening but may be temporarily incapacitating. Examples are simple bone fractures or major abrasions [14]. Fatalities are not measured on this scale. Using actual crash data, Hitchcock related AIS values to relative velocity at impact [15]. For crashes at

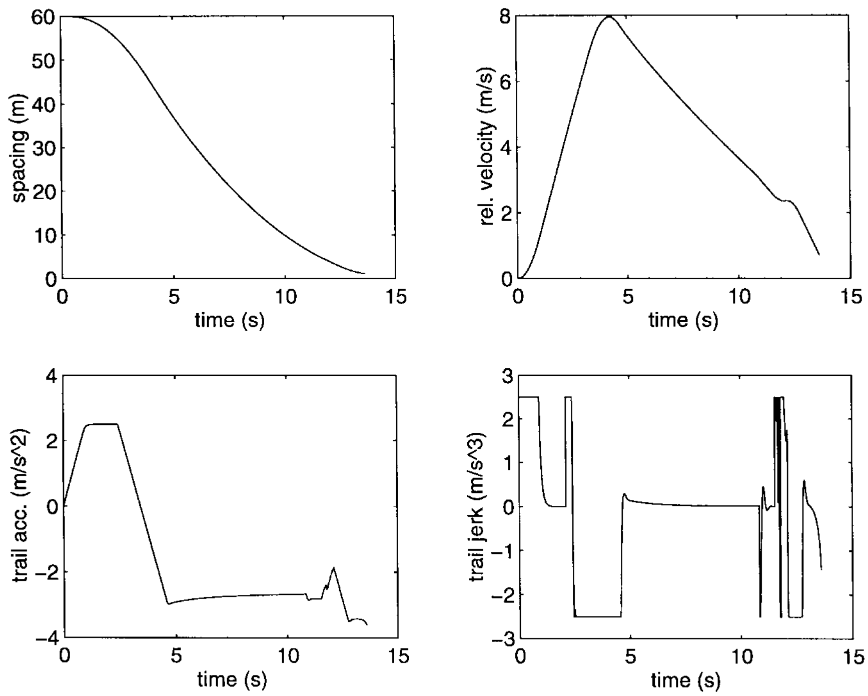

Fig. 6. Simulation results of join from 60-m initial spacing. The initial velocity of both lead and trail platoons was 25 m/s. The lead platoon applied comfort braking at 4.1 s.

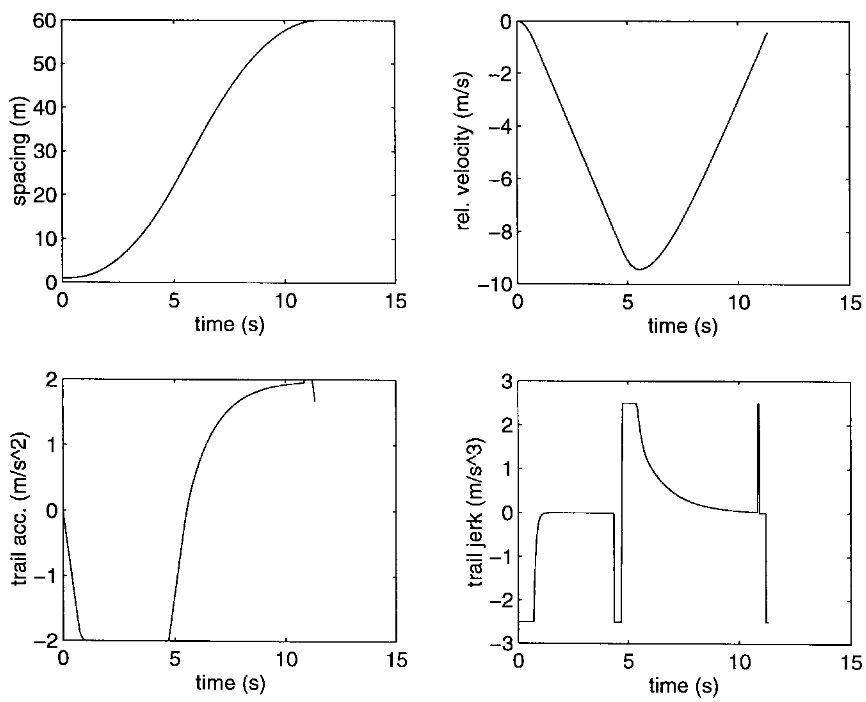

Fig. 7. Simulation results of split from 1–60-m spacing. The initial velocity of the original platoon was 25 m/s.

or below 3.3 m/s, he found no probability of fatalities or injuries rated AIS ≥ 3. The probability of injuries rated AIS $= 2$ at that speed or slower is low.

7) $\Delta x_{\text{join}} = 1$ m. This is the current intraplatoon spacing.
8) $\Delta x_{\text{split}} = \Delta x_{\text{leader}} = \Delta x_{\text{change}} = 60$ m. This is the current interplatoon distance.
9) $d = 30$ ms. Simple brake models often include pure time delays of about 50 ms. It is shown in [16], however, that delays in the current braking system for PATH are greater than 150 ms. By redesigning the brake system, delays near 20 ms could be achieved [17]. Delays from sensing, filtering and differentiating are also possible, but they could be small at a high sample rate. The sample time used in the simulations shown here was 10 ms.
10) $\lambda_1 = 0.6$, $\lambda_2 = 15$, $\beta = 3.9$, $L_1 = 1$, $L_2 = 15$, $\gamma = 1.1$ and $e_\infty = 0.30$ m/s. These values were determined

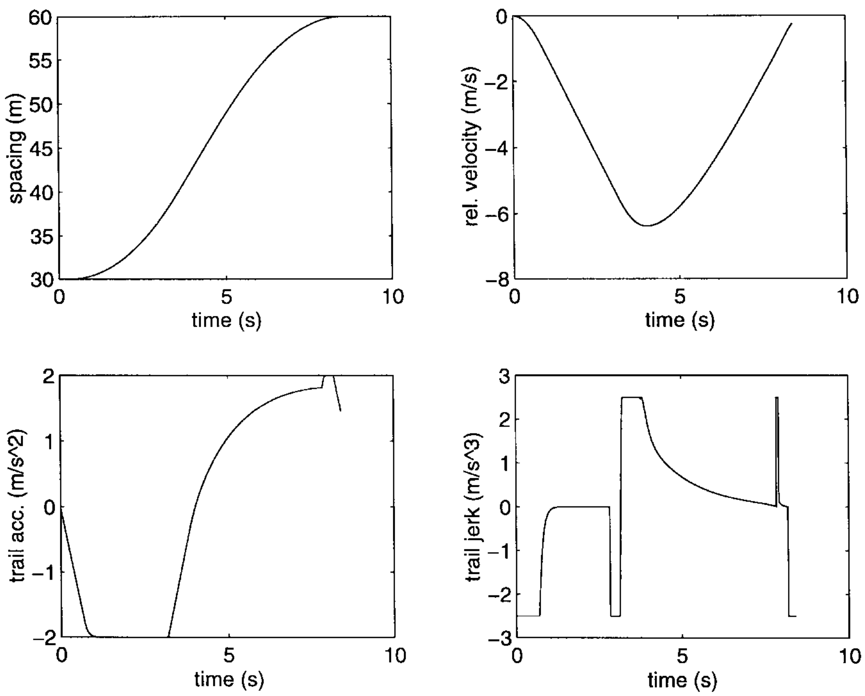

Fig. 8. Simulation results of split from 30–60-m spacing. The initial velocity of the original platoon was 25 m/s.

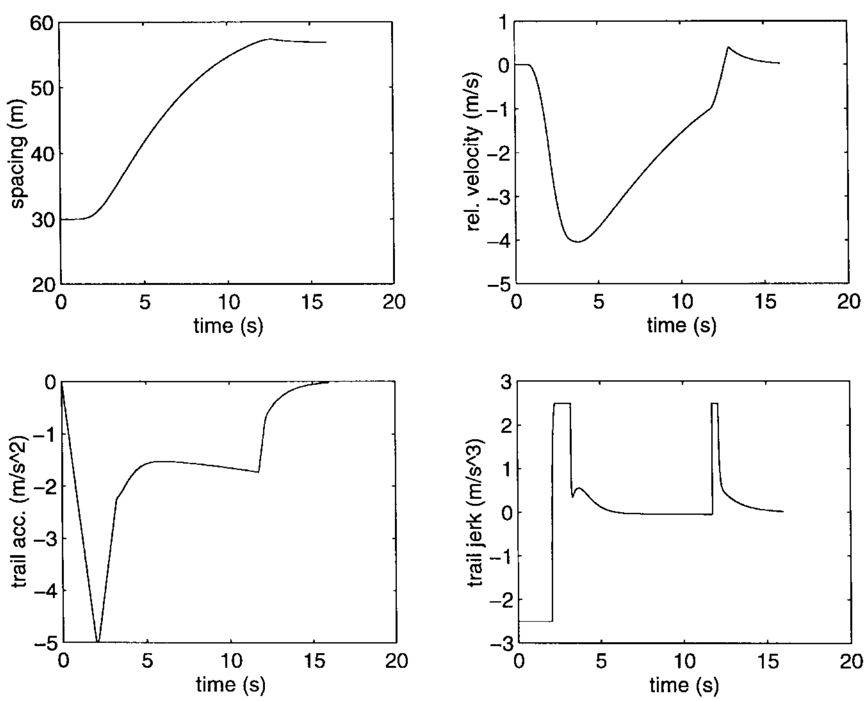

Fig. 9. Simulation results of split from 30 m spacing: The lead platoon is applying comfort braking from an initial velocity of 25 m/s.

by examining the error behavior of the controller. The procedure for their calculation is described in [6].

Fig. 3 shows results for a join from 30-m initial spacing. The velocity of the platoon ahead was constant at 25 m/s. The maneuver was completed in 11.8 s. Jerk and acceleration comfort limits were not exceeded. The final relative velocity is not zero as the simulation only ran to the point where the follower law takes effect.

Fig. 4 shows results from a join with an initial spacing of 60 m. The lead platoon maintained a constant velocity. The join took 16.5 s in this case.

Fig. 5 show the case in which the lead platoon applies maximum braking when the trail platoon has maximum relative velocity. The simulations were allowed to run until the trail platoon either stopped or collided. Note that the simulation shows a collision but, as expected, the impact speed was lower than $v_{\rm allow}$. The figures include a large spike in jerk. The

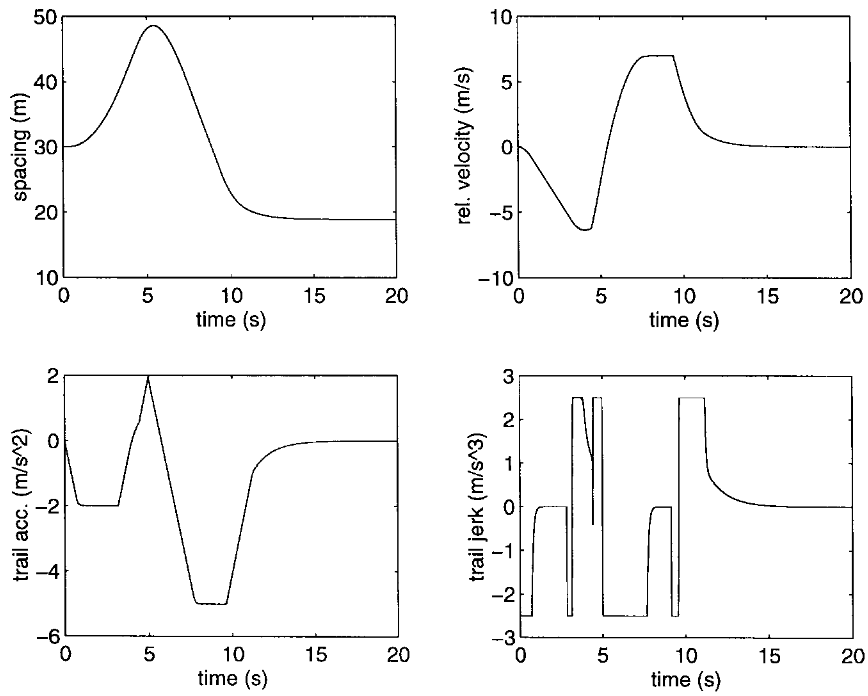

Fig. 10. Simulation results of split from 30 m spacing: The lead platoon is applying full braking from an initial velocity of 25 m/s. $v_{\min} = 0$ m/s.

controller is designed so that comfort limits are disregarded when safety becomes critical. In these cases, the saturation function on jerk was overridden once the large lead platoon deceleration was detected.

In the final join simulation, the lead platoon braked at comfortable deceleration. No collision occurred. The results are shown in Fig. 6.

The split law was also simulated. Figs. 7 and 8 show the results of split from 1- and 30–60-m spacing. The cases when the lead platoon apply comfort and full braking while the trail platoon is attempting a split are shown in Figs. 9 and 10, respectively.

The results of the other control laws are not shown, since they are similar to the ones already presented.

VI. Conclusions

In this paper the notion of safe longitudinal control law is introduced. By using this notion, the control strategy proposed in this paper improves the safety and preserves the comfort of the platoon leader longitudinal control laws. A state space desired velocity profile for the platoon is used in place of the timed trajectories used in other designs. The profile guarantees minimum time performance with comfort, whenever safety is not compromised. This strategy also increases the robustness of the maneuvers to variable acceleration performances of the platoon ahead. By changing parameters in the control equations the control laws can also be applied in cases of bad weather or degraded road conditions.

When the state of the platoon is in the safe region defined in this paper, transitions between different control laws are always safe. The control laws reduce the threat of high-speed collisions between platoon. The major cost of the added safety is the extra time that maneuvers can take to complete, particularly when current braking delays are taken into account. The cost of ensuring a comfortable ride during the split and decelerate to change lane maneuvers is also time, but to a lesser degree than for the safety requirements.

The designed nonlinear velocity tracking controller keeps the velocity of the platoon within a given error bound, while depending only in the information currently available in the PATH architecture.

Increments in the time needed to complete the longitudinal maneuvers reduce the time available for other maneuvers and ultimately limit the capacity of the automated highway [18]. If these increments cause large reductions in capacity, alternatives to the actual longitudinal maneuvers will have to be considered more carefully.

This controller should be tested in conjunction with a realistic car model. Inputs that accurately reflect sensor readings also must be added to the model. The control strategy must also be tested for its ability to handle degraded sensor operation or sensor failure.

Appendix
Proof of Theorem 2.1

Proof: Let v_{allow} be the maximum allowable relative impact speed and d be the delay for maximum deceleration to be achieved when a maximum braking command is applied. Define the set $X_{\text{safe}} \subset \Re^3$ to be the set of $(\Delta x, \Delta\dot{x}, v_{\text{lead}})$ that satisfy the equation shown at the bottom of the following page. Denote by $X_{\text{bound}} \subset \Re^3$ the set of $(\Delta x, \Delta\dot{x}, v_{\text{lead}})$ that satisfy

$$-\Delta\dot{x} < \max(-v_{\text{lead}} + \sqrt{2a_{\min}\Delta x + v_{\text{lead}}^2 + v_{\text{allow}}^2}, v_{\text{allow}}).$$

Consider a control that would apply maximum braking whenever $(\Delta x, \Delta\dot{x}, v_{\rm lead}) \notin X_{\rm safe}$. By assumption, the trail platoon will decelerate at $-a_{\rm min}$ d seconds after the maximum braking is applied. If maximum braking is applied at time t, the acceleration of the trail platoon at time $\tau \in [t, t+d]$ can take values in $[-a_{\rm min}, a_{\rm max}]$.

Suppose that $(\Delta x(0), \Delta\dot{x}(0), v_{\rm lead}(0)) \in X_{\rm safe}$, we will show that under the proposed control law, $(\Delta x(t), \Delta\dot{x}(t), v_{\rm lead}(t)) \in \overline{X_{\rm bound}}$ for all $t \geq 0$. Firstly, notice that since $X_{\rm safe} \in \overline{X_{\rm bound}}$, this is true if $(\Delta x(t), \Delta\dot{x}(t), v_{\rm lead}(t)) \in X_{\rm safe}$ for all $t \geq 0$. Because $(\Delta x(t), \Delta\dot{x}(t), v_{\rm lead}(t))$ is continuous in t, if $(\Delta x(t), \Delta\dot{x}(t), v_{\rm lead}(t)) \notin X_{\rm safe}$ for some time $t > 0$, then there exists T_1, $t \geq T_1 > 0$ when $(\Delta x(T_1), \Delta\dot{x}(T_1), v_{\rm lead}(T_1))$ lies on the boundary of $X_{\rm safe}$. For $t \in [T_1, T_1+d]$

$$\Delta x(t) = \int_{T_1}^{t}\int_{T_1}^{\tau} w(\sigma)\, d\sigma\, d\tau - \int_{T_1}^{t}\int_{T_1}^{\tau} u_a(\sigma)\, d\sigma\, d\tau + \Delta\dot{x}(T_1)(t-T_1) + \Delta x(T_1).$$

Consider the following function that is the separation of $\Delta\dot{x}$ from the velocity boundary of $X_{\rm bound}$

$$g(\Delta x, v_{\rm lead}, v_{\rm trail}) := \Delta\dot{x} - b_1(\Delta x, \Delta\dot{x}, v_{\rm lead})$$

where

$$\begin{aligned} &b_1(\Delta x, \Delta\dot{x}, v_{\rm lead}) \\ &\quad := \min(v_{\rm lead}(t) \\ &\qquad - \sqrt{2a_{\rm min}\Delta x(t) + v_{\rm lead}^2(t) + v_{\rm allow}^2}, -v_{\rm allow}) \end{aligned}$$

is the relative velocity boundary of the set $X_{\rm bound}$ and $\Delta\dot{x} = v_{\rm lead} - v_{\rm trail}$. Hence, for $\Delta x \geq 0$, the triple $(\Delta x, \Delta\dot{x}, v_{\rm lead}) \in X_{\rm bound}$ if and only if $g(\Delta x, v_{\rm lead}, v_{\rm trail}) \geq 0$ and $\Delta x \geq 0$. Notice that for $\epsilon \in \Re$

$$\operatorname{sign}(\epsilon)[g(\Delta x, v_{\rm lead}+\epsilon, v_{\rm trail}) - g(\Delta x, v_{\rm lead}, v_{\rm trail})] \geq 0 \tag{31}$$

$$\operatorname{sign}(\epsilon)[g(\Delta x, v_{\rm lead}, v_{\rm trail}+\epsilon) - g(\Delta x, v_{\rm lead}, v_{\rm trail})] \leq 0 \tag{32}$$

$$\operatorname{sign}(\epsilon)[g(\Delta x+\epsilon, v_{\rm lead}, v_{\rm trail}) - g(\Delta x, v_{\rm lead}, v_{\rm trail})] \geq 0 \tag{33}$$

From relationships (31)–(33), for any $t \in [T_1, T_1+d]$, $g(\Delta x(t), v_{\rm lead}(t), v_{\rm trail}(t))$ is minimized if $\Delta x(t)$ and $v_{\rm lead}(t)$ are minimized, and $v_{\rm trail}(t)$ is maximized. Since $w(t), u_a(t) \in [-a_{\rm min}, a_{\rm max}]$, these three conditions are simultaneously achieved if $w(t) = -a_{\rm min}$, $u_a(t) = a_{\rm max}$ for all $t \in [T_1, T_1+d]$. Define for $t \in [T_1, T_1+d]$

$$\begin{aligned} \overline{v}_{\rm lead}(t) &:= v_{\rm lead}(T_1) - (t-T_1)a_{\rm min} \\ \overline{v}_{\rm trail}(t) &:= v_{\rm trail}(T_1) + (t-T_1)a_{\rm max} \\ \overline{\Delta x}(t) &:= -\frac{(t-T_1)^2}{2}(a_{\rm min}+a_{\rm max}) \\ &\qquad + \Delta\dot{x}(T_1)(t-T_1) + \Delta x(T_1). \end{aligned}$$

Thus, for $t \in [T_1, T_1+d]$

$$\begin{aligned} g(\Delta x(t), v_{\rm lead}(t), v_{\rm trail}(t)) &\geq g(\overline{\Delta x}(t), \overline{v}_{\rm lead}(t), \overline{v}_{\rm trail}(t)) \\ &\geq g(\overline{\Delta x}(d), \overline{v}_{\rm lead}(d), \overline{v}_{\rm trail}(d)). \end{aligned}$$

We will show that $\overline{g} := g(\overline{\Delta x}(T_1+d), \overline{v}_{\rm lead}(T_1+d), \overline{v}_{\rm trail}(T_1+d)) \geq 0$.

At $t = T_1$, since $(\Delta x(T_1), \Delta\dot{x}(T_1), v_{\rm lead}(T_1))$ is on the boundary of $X_{\rm safe}$, we have either (34), shown at the bottom of the page, or

$$v_{\rm trail}(T_1) = -(a_{\rm max}+a_{\rm min})\, d + v_{\rm allow} + v_{\rm lead}(T_1). \tag{35}$$

The bound $\overline{g} := g(\overline{\Delta x}(T_1+d), \overline{v}_{\rm lead}(T_1+d), \overline{v}_{\rm trail}(T_1+d))$ is given by

$$\begin{aligned} \overline{g} = \Delta\dot{x}(T_1) - (a_{\rm min}+a_{\rm max})\, d - \min(&v_{\rm lead}(T_1) - a_{\rm min}\, d \\ &- \sqrt{2a_{\rm min}\overline{\Delta x}(T_1+d) + \overline{v}_{\rm lead}^2(T_1+d) + v_{\rm allow}^2} \\ &- v_{\rm allow}). \end{aligned}$$

Suppose that (34) is true, then we have the following equation, shown at the top of the following page. If, on the other hand (35) is true, then

$$\begin{aligned} \overline{g} &= v_{\rm lead}(T_1) - v_{\rm trail}(T_1) - (a_{\rm min}+a_{\rm max})\, d + v_{\rm allow} \\ &= v_{\rm lead}(T_1) - (v_{\rm lead}(T_1) + v_{\rm allow} \\ &\quad - (a_{\rm min}+a_{\rm max})\, d) + -(a_{\rm min}+a_{\rm max})\, d + v_{\rm allow} = 0. \end{aligned}$$

$$-\Delta\dot{x} < \max\begin{cases} -(a_{\rm max}+a_{\rm min})\, d - v_{\rm lead} \\ \quad + \sqrt{2a_{\rm min}\Delta x + v_{\rm lead}^2 + v_{\rm allow}^2 + a_{\rm min}(a_{\rm max}+a_{\rm min})d^2} \\ -(a_{\rm max}+a_{\rm min})\, d + v_{\rm allow}. \end{cases}$$

$$v_{\rm trail}(T_1) = -(a_{\rm max}+a_{\rm min})\, d + \sqrt{2a_{\rm min}\Delta x(T_1) + v_{\rm lead}^2(T_1) + v_{\rm allow}^2 + a_{\rm min}(a_{\rm max}+a_{\rm min})\, d^2} \tag{34}$$

Thus, if either (34) or (35) is true, then for any $t \in [T_1, T_1+d]$

$$g(\Delta(t), v_{\text{lead}}(t), v_{\text{trail}}(t)) \geq \overline{g} \geq 0.$$

For $t \geq T_1 + d$, full braking is achieved i.e., $u_a(t) = -a_{\min}$.

We now show that if $g(T_1 + d) \leq 0$, then $g(t) \leq 0$ for all $t \geq T_1 + d$. From relationships (31) and (33), $g(\Delta(t), v_{\text{lead}}(t), v_{\text{trail}}(t))$ is minimized if $v_{\text{lead}}(t)$ and $\Delta x(t)$ are minimized. This is achieved if $w(t) = -a_{\min}$ for $t \in [T_1 + d, \inf)$ or until $v_{\text{lead}}(t) = 0$.

Under this worst case scenario, for the first choice in the argument of $g(t)$ is shown in the equation at the middle of the following page, but as $v_{\text{trail}} \geq \sqrt{2a_{\min}\Delta x(T_1+d) + v_{\text{lead}}^2(T_1+d) + v_{\text{allow}}^2}$ when $(\Delta x, \Delta\dot{x}, v_{\text{lead}}) \notin X_{\text{bound}}$ as shown in the equation at the bottom of the page, or, for the second choice in the argument of $g(t)$

$$g(t) = \Delta\dot{x}(T_1+d) + v_{\text{allow}} = g(T_1+d) \leq 0.$$

$$\begin{aligned}
\overline{g} =& v_{\text{lead}}(T_1) - v_{\text{trail}}(T_1) - (a_{\max} + a_{\min})\, d - v_{\text{lead}}(T_1) + a_{\min}\, d \\
&+ \sqrt{2a_{\min}\overline{\Delta x}(T_1+d) + \overline{v}_{\text{lead}}^2(T_1+d) + v_{\text{allow}}^2} \\
=& (a_{\max} + a_{\min})\, d - \sqrt{2a_{\min}\Delta x(T_1) + v_{\text{lead}}^2(T_1) + v_{\text{allow}}^2 + a_{\min}(a_{\max} + a_{\min})\, d^2} \\
&- (a_{\max} + a_{\min})\, d + \sqrt{2a_{\min}\overline{\Delta x}(T_1+d) + \overline{v}_{\text{lead}}^2(T_1+d) + v_{\text{allow}}^2} \\
=& -\sqrt{2a_{\min}\Delta x(T_1) + v_{\text{lead}}^2(T_1) + v_{\text{allow}}^2 + a_{\min}(a_{\max} + a_{\min})\, d^2} + a_{\min}\, d \\
&+ \sqrt{-2a_{\min}\left(\frac{d^2}{2}\,(a_{\min} + a_{\max}) - \Delta\dot{x}(T_1)\, d - \Delta x(T_1)\right) + (v_{\text{lead}}(T_1) - a_{\min}\, d)^2 + v_{\text{allow}}^2} \\
=& -\sqrt{2a_{\min}\Delta x(T_1) + v_{\text{lead}}^2(T_1) + v_{\text{allow}}^2 + a_{\min}(a_{\max} + a_{\min})\, d^2} + a_{\min}\, d \\
&- \sqrt{\left(\sqrt{2a_{\min}\Delta x(T_1) + v_{\text{lead}}^2(T_1) + v_{\text{allow}}^2 + a_{\min}(a_{\max} + a_{\min})\, d^2} - a_{\min}\, d\right)^2} \\
=& -\sqrt{2a_{\min}\Delta x(T_1) + v_{\text{lead}}^2(T_1) + v_{\text{allow}}^2 + a_{\min}(a_{\max} + a_{\min}) d^2} + a_{\min}\, d \\
&+ \sqrt{2a_{\min}\Delta x(T_1) + v_{\text{lead}}^2(T_1) + v_{\text{allow}}^2 + a_{\min}(a_{\max} + a_{\min})\, d^2} - a_{\min}\, d = 0.
\end{aligned}$$

$$\begin{aligned}
g(t) =& \Delta\dot{x}(T_1+d) - v_{\text{lead}}(T_1+d) + a_{\min}(t - T_1 + d) + \sqrt{2a_{\min}\Delta x(t - T_1 + d) + v_{\text{lead}}^2(t - T_1 + d) + v_{\text{allow}}^2} \\
=& \Delta\dot{x}(T_1+d) - v_{\text{lead}}(T_1+d) + a_{\min}(t - T_1 + d) \\
&+ \sqrt{\begin{aligned}&2a_{\min}\Delta x(T_1+d) + 2a_{\min}\Delta\dot{x}(T_1+d)(t - T_1 + d) + v_{\text{lead}}^2(T_1+d) \\ &- 2a_{\min}(t - T_1 + d)v_{\text{lead}}(T_1+d) + a_{\min}^2(t - T_1 + d)^2 + v_{\text{allow}}^2\end{aligned}} \\
=& \Delta\dot{x}(T_1+d) - v_{\text{lead}}(T_1+d) + a_{\min}(t - T_1 + d) \\
&+ \sqrt{2a_{\min}\Delta x(T_1+d) + v_{\text{lead}}^2(T_1+d) + v_{\text{allow}}^2 - 2a_{\min}v_{\text{trail}}(T_1+d)(t - T_1 + d) + a_{\min}^2(t - T_1 + d)^2}
\end{aligned}$$

$$\begin{aligned}
g(t) \leq& \Delta\dot{x}(T_1+d) - v_{\text{lead}}(T_1+d) + a_{\min}(t - T_1 + d) \\
&+ \sqrt{\begin{aligned}&2a_{\min}\Delta x(T_1+d) + v_{\text{lead}}^2(T_1+d) + v_{\text{allow}}^2 - 2a_{\min}(t - T_1 + d) \\ &\sqrt{2a_{\min}\Delta x(T_1+d) + v_{\text{lead}}^2(T_1+d) + v_{\text{allow}}^2} + a_{\min}^2(t - T_1 + d)^2\end{aligned}} \\
=& \Delta\dot{x}(T_1+d) - v_{\text{lead}}(T_1+d) + a_{\min}(t - T_1 + d) \\
&+ \sqrt{\left(\sqrt{2a_{\min}\Delta x(T_1+d) + v_{\text{lead}}^2(T_1+d) + v_{\text{allow}}^2} - a_{\min}(t - T_1 + d)\right)^2} \\
=& \Delta\dot{x}(T_1+d) - v_{\text{lead}}(T_1+d) + \sqrt{2a_{\min}\Delta x(T_1+d) + v_{\text{lead}}^2(T_1+d) + v_{\text{allow}}^2} = g(T_1+d) \leq 0
\end{aligned}$$

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