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CONTROL OF SELF-OPTIMIZING EXERCISE MACHINES

Roberto Horowitz *,[1] **Perry Y. Li** **,[2] **Joel Shields** *,[3]

* *Department of Mechanical Engineering, University of California, Berkeley CA 94720-1740.*
** *Department of Mechanical Engineering, University of Minnesota, Minneapolis, MN 55455*

Abstract. The control of a one degree of freedom exercise machine is considered. The control objective consists in making the human user exercise in a manner that maximizes his consumption of power. The optimality condition is determined by the muscle mechanics which is assumed to satisfy a force-position-velocity relationship. In general, the parameters of this relationship are unknown and vary with the configuration of the exercise machine. As a consequence, the control scheme must simultaneously i) identify the user's strength characteristic, ii) optimize the controller, and iii) stabilize the system to the estimated optimal state. In this paper we present control systems in the form of a nonlinear dynamic or static dampers that make the controlled system interact passively with the user. Adaptive and self-optimizing control strategies are discussed, which achieve the control objectives described above. Results of a clinical study are presented which corroborate many of the assumptions used in this paper and verify the efficacy of the proposed control schemes.

Keywords. adaptive control, self-optimization, dual control, robotics, passivity, biomechanics, motion control.

1. INTRODUCTION

In this paper we discuss the controller design, implementation and experimental verification of a new class of self-optimizing exercise machines that are capable of providing the user with an ***optimal*** workout. These control systems achieve this objective by: i) identifying the strength characteristics of its user; ii) determining the optimal exercise routine for that particular user; iii) controlling the exercise machine so that the user actually performs the optimal workout and iv) interacting safely with the user by imposing that the direction of net energy flow is always from the user to the exercise machine.

The type of exercise machines that are consider in this paper are those whose resistance is based on dampers; whose motion consists of a single degree of freedom, repetitive motion and whose objective is to provide the user with a cardiovascular workout and caloric consumption. The control objective consists in maximizing the instantaneous power produced by the user at all times during the exercise workout, for a given user constant effort level. In other words, the machine will attempt to make the user consume the maximum possible amount of calories, for a given effort level, by the end of the workout. We will call this type of exercise regime an *opti-poteric workout* [4].

To successfully design a control system that satisfies all of the above requirements, several mechatronics issues

[1] Email: `horowitz@me.berkeley.edu`
[2] Email: `pli@me.umn.edu`
[3] Email: `jfshield@scorpio.me.berkeley.edu`

[4] From the roots *opti-* meaning maximum, and *poter-* which is late latin for power.

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must be addressed. The first is how to effectively describe the task that must be executed by the exercise machine. Traditionally, robot manipulation and many other mechatronic tasks are specified by parameterizing the desired state or output of the system as a function of time. However, in the case of exercise machines, opti-poteric workouts are best represented as desired velocity fields, i.e. the assignment of a desired velocity for every position in the configuration space of the exercise machine. As a consequence, the control task will consist in tracking an optimal velocity field.

A second issue concerns the safety of the user. To guarantee that the direction of net energy flow is always from the user to the exercise machine, we will impose that the exercise machine remains *closed loop passive* with respect to the input/output pair formed by the generalized forces provided by the user and the generalized velocity of the exercise machine. Passivity is a well known property of a class of dynamic systems, which has been extensively used in the design of adaptive and non-adaptive control systems for mechatronic devices, cf. (Slotine and Li, 1991; ZODIAC, 1996). However, most so-called passivity based control systems do not preserve under feedback control the passivity of the control systems with respect to the input/output pair formed by the external inputs and the system output. In this paper we will utilize the *passive velocity field control* approach, originally introduced in (Li and Horowitz, 1995), to formulate a controller that tracks a prescribed velocity field and maintains the closed loop passivity of the exercise machine with respect to the external forces provided by the user (Li and Horowitz, 1997*a*). This controller has the additional advantage that it can be implemented using only passive mechanical elements such as a linear spring and a set of four adjustable dampers. Unfortunately, most exercise machines on the market today are equipped with only static dampers, and the complete passive velocity field control scheme cannot be implemented with only one static damper. To account for this limitation, we will also present a simplified velocity field tracking control scheme that only utilizes a static damper, at the expense of sacrificing perfect tracking (Shields and Horowitz, 1997).

The third issue that must be accounted for in the control system design for a smart exercise machine is the fact that the opti-poteric velocity field is a function of the user's biomechanics, which may not be known a-priori. Thus, the smart exercise machine controller must be capable of identifying the user's biomechanics characteristics, in order to subsequently determine th opti-poteric desired velocity field, which the exercise machine must track. Because the desired velocity field cannot be defined a-priori, since it depends on the unknown user's biomechanics which may vary as the user fatigues, the control system has to be able to decide when it must identify the user's biomechanics and when it should track its estimate of the opti-poteric desired velocity field. This problem was first considered in (Li and Horowitz, 1997*b*), and a *self optimizing control strategy* was formulated and implemented to solve it.

In order to satisfy all of the design constraints discussed above, the control schemes that are presented in this paper are formulated using the following design methodology: (1) A controller capable of causing the user to exercise according to an arbitrary velocity profile is first developed, assuming that the user's muscular biomechanic characteristics are known. The closed loop system is constrained to be passive, in order to satisfy safety requirements. (2) A certainty equivalence adaptive control scheme, based on the controller developed in step 1, is then formulated to identify the user's muscular biomechanic characteristics and control the exercise system. (3) A self-optimizing supervisor automaton, which consists of a reference generator and a finite state machine supervisor, is then constructed to guarantee that an accurate optimal velocity field is determined and followed by the adaptive controller developed in step 2. Due to space constraints, we will not discuss in detail the self-optimizing strategy in step 3 in this paper. Interested readers are referred to (Li and Horowitz, 1997*b*).

In section 2 we briefly discuss several biomechanic factors that affect the performance of a user during an exercise and formulate the self-optimizing exercise machine control problem. Section 3 describes the formulation of a dynamic damping control scheme which guarantees the closed loop passivity of the exercise system and tracks an arbitrary velocity field, assuming that the human forces are known. A simplified static damping control scheme, which guarantees under similar assumptions closed loop passivity and the ultimate boundedness of the velocity field tracking error is described in section 4. Section 5 describes the formulation of certainty equivalence adaptive controllers which identify the unknown human Hill surface force-velocity-position relation, and achieve tracking of an arbitrary velocity field. Section 6 briefly describes a self-optimizing strategy for simultaneous identification of the Hill surface strength parameters and the optimal velocity field. Results of a clinical study are presented in section 7 that verified the effectiveness of the proposed control strategy in identifying the optimal velocity field and controlling the exercise machine to achieve an optimal workout. Conclusions are given in section 8.

2. BIOMECHANICS AND PROBLEM FORMULATION

The most important factors that affect the force that a muscle exerts on the exercise machine are (McMahon, 1984; Lieber, 1992): *(1) The user's effort level:* Muscle activation is under voluntary control and to a lesser extent depends on the reflexes. *(2) The exerise machine's configuration:* As the configuration of the exercise machine varies, the leverages that the muscle forces have on the machine changes. The net effect is that the generalized force exerted by the user on the exercise machine depends nonlinearly on the generalized position of the exercise motion. *(3) The velocity of motion:* It has been found empirically that for a given level of electrical stimulation, the force produced by a muscle decreases monotonically with the rate of shortening of the muscle (Hill, 1938). A hyperbolic relation has been found to be a good approximation of this relationship over a large range of velocities (Fig. 1). It is believed that this is due to the asymmetric rate of cross-bridge attachment and detachment as the muscle shortens. *(4)*

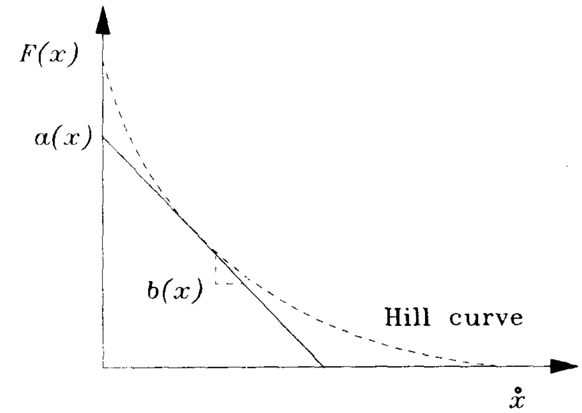

Fig. 1. Force-velocity (Hill) relation of an activated muscle

The user's fatigue level: The force exerted on the exercise machine decreases as the user fatigues, even under a constant effort level. The mechanism of fatigue is still an active area of research in muscle physiology. Most exercise motions involve more than one joint, and each joint is activated by more than one muscle. We shall group all the muscles that actuate the exercise motion, and refer to them collectively as the ***equivalent muscle***. For a given user effort level and fatigue state, the generalized force that the equivalent muscle exerts on the exercise machine is a nonlinear function of position, and it decreases monotonically with the velocity of the exercise motion. We will describe this characteristic by a generalized force-position-velocity relation (Fig. 2), which we will call the Hill surface, in honor or A. Hill (Hill, 1938) who first proposed the force-velocity relation for a single activated muscle shown in Fig. 1.

Given the factors that affect muscle force discussed above, it is possible to define the opti-poteric control strategy, which maximizes the instantaneous power generated by the user and takes into account some of these factors. Since the level of muscle activation (effort level) is voluntary, we assume that the user maintains a constant effort level while he exercises. As discussed in section 7,

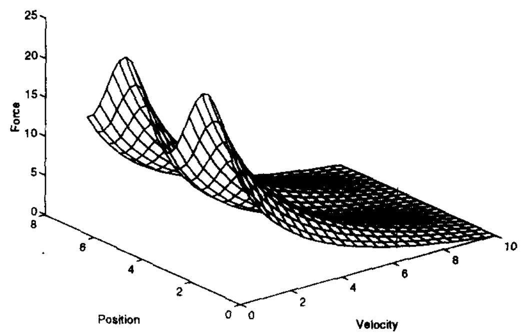

Fig. 2. A typical Hill surface

the user can be aided by bio-feedback methods or can be instructed and trained to exercise in this manner.

We now define the instantaneous power generated by the user

$$J(F, \dot{x}) = F\,\dot{x}\,, \tag{1}$$

which is to be maximized. In (1), F and $\dot{x}$ are respectively the equivalent muscle's generalized force and the generalized velocity, which are related by the Hill (force-velocity) relation. Because of the monotonic property of the Hill relation, $\max J(\cdot,\cdot)$ exists at every instant of time. Hence an efficient exercise is achieved since (mechanical) energy is being consumed at the maximum rate in this exercise.

Without loss of generality, we assume that the exercise motion has been designed and consists of a one degree of freedom repetitive movement. We will assume that the configuration space $\mathcal{G}$ is the circle S^1. We also assume that the resistive force to movement can be manipulated in real time.

2.1 *Arm Cranking Experimental Setup*

The experimental exercise machine that was used in some of our research is schematically depicted in figure 3. It consists of a NSK series 3 D.C. motor, which

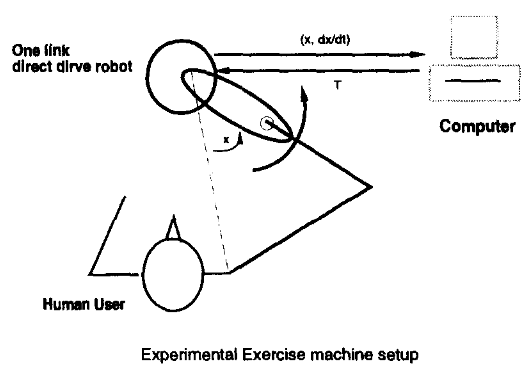

Fig. 3. Experimental exercise machine setup.

is torque rated at 40Nm, and a 25cm long rigid link. A handle is mounted on the end of the link via a bearing. The user sits next to the setup and exercises by turning the one link robot while holding onto the handle. The rigid link, the forearm, the upperarm and the shoulder of the user are assumed to lie on the horizontal plane. In reality, the shoulder is generally approximately 10cm above the rigid link. The shoulder is assumed fixed so that the kinematics is that of a four-bar linkage. Since the elbow of the user is constrained, the angle of the rigid link relative to a point on the base of the motor specifies the configuration uniquely.

The dynamics of the exercise system takes the form:

$$M(x(t))\ddot{x}(t) + C(x(t))\dot{x}^2(t) = F(t) + T(t), \quad (2)$$

where $M(x) \in \Re^+$ is the generalized inertia of the four bar linkage, $C(x)\dot{x}^2$ represents the Coriolis and centripetal forces, F and T are the generalized forces generated by the user's equivalent muscle and the motor respectively. We assume that the function $M(\cdot)$ (and thus $C(\cdot)$) is known. The inertia of the user's limbs can be found from regression curves developed from databases (e.g. (Zatsiorsky and Seluyanov, 1983)) or, alternatively, $M(\cdot)$ can be obtained from the short burst experiments similar to those conducted by (Lehman and Calhoun, 1990).

In a multi-muscle actuated motion, the generalized forces of the muscles add ($\sum_g F_g$) but the hyperbolic Hill force-velocity relation depicted in Fig 1 does not strictly hold for the total force. In our application, we use an affine approximation of the hyperbolic relation for each muscle group. When the affine approximation is derived for the overall force, the processes of taking the linear approximation and of adding the forces over multiple muscle groups commute. Thus, we make the assumption that for an effort level and a fatigue state, the total equivalent muscle force F in Eq. (2) is given by,

$$F = F_h(x, \dot{x}) = a(x) - b(x)\dot{x}, \quad (3)$$

where the argument x in the positive functions $a(\cdot)$ and $b(\cdot)$ expresses the dependence of the force-velocity curves on the geometry of the muscle with respect to the configuration of the exercise motion. Both $a(\cdot)$ and $b(\cdot)$ generally depend on the effort level and the fatigue state of the muscle.

Assuming that (3) is satisfied, the optimal condition at each x is given by:

$$V^*(x) = \frac{1}{2}\frac{a(x)}{b(x)}, \qquad F^*(x) = \frac{a(x)}{2}. \quad (4)$$

The control objectives are the following:

(1) Cause the velocity $\dot{x}$ to follow the optimal velocity field V^*, i.e. $\dot{x}(t) \to V^*(x(t))$.

(2) Since the exercise machine is to interact with the user mechanically, we also require, for safety reasons, that the controlled exercise machine behave passively with respect to the user, so that the user will not absorb more energy than what he / she puts in. By this we mean that the velocity $\dot{x}$, and the user's force $F(t)$ must satisfy the following passivity relation,

$$\int_0^t F(\tau)\dot{x}(\tau)d\tau \geq -c^2. \quad (5)$$

for all $t \geq 0$ and any human force $F(\tau)$.

3. DYNAMIC DAMPING CONTROL

In this section, we develop a passive controller assuming that the human force F in Eq. (2) is known. This assumption will be removed in section 5.3, when a certainty equivalence adaptive controller is introduced. As shown in (Li, 1995), it is not possible to design a static damping controller which will track an arbitrary desired velocity field and make the closed loop exercise control system passive with respect to the user. The basic difficulty lies on the fact that the amount of power that a static damping controller can manipulate is limited by the instantaneous power supplied by the user while executing the desired exercise. Thus, to achieve both the tracking and passivity objectives simultaneously, it is necessary to design a dynamic passive controller, using the velocity field design methodology introduced in (Li and Horowitz, 1995). The basic idea is to incorporate in the control structure the dynamics of an energy storage element which mimics either a spring or a flywheel. The controller is constructed in two steps. First, we ignore the interaction with the user, and construct a passive velocity field controller capable of tracking the desired velocity field when sufficient energy is stored in the system. Secondly, the interaction with the user is taken into account by designing a damper that dissipates all the power supplied by the user when the exercise closed loop system is tracking the desired velocity field.

3.1 *Passive Velocity Field Control*

As discussed above, in order to design a controller which is capable of tracking an arbitrary desired velocity field and make the closed loop exercise control system passive with respect to the user, it is necessary to introduced in the control structure an additional energy storage element. For the purpose of formulating the control algorithm, it is convenient to think of this element as a fictitious flywheel, with dynamics given by

$$M_f \dot{v}_f = T_f, \quad (6)$$

where M_f and v_f are respectively interpretated as the inertia and angular velocity of the flywheel. The control

T_f will be determined subsequently, as part of the control law for the augmented system. Alternatively, one can think of $1/M_f$ as the stiffness coefficient of a mechanical spring and of v_f as the compressive force in the spring. This realization is convenient for implementing the control law using only passive mechanical elements, as will be detailed in section 3.3.

Combining the exercise machine control system dynamics given by (2) and the dynamics of a fictitious flywheel results in the dynamics of an augmented mechanical system which is given by

$$\mathbf{M}_a(x)\dot{\mathbf{v}}_a + \mathbf{C}_a(x,\dot{x})\mathbf{v}_a = \mathbf{T}_a + \mathbf{F}_a \qquad (7)$$

where

$$\mathbf{M}_a(x) = \begin{pmatrix} M(x) & 0 \\ 0 & M_f \end{pmatrix}, \quad \mathbf{C}_a(x,\dot{x}) = \begin{pmatrix} C(x)\dot{x} & 0 \\ 0 & 0 \end{pmatrix} \qquad (8)$$

$$\mathbf{T}_a = \begin{pmatrix} T \\ T_f \end{pmatrix}, \quad \mathbf{F}_a = \begin{pmatrix} F \\ 0 \end{pmatrix}, \text{ and } \mathbf{v}_a = \begin{pmatrix} \dot{x} \\ v_f \end{pmatrix},$$

The total energy of the augmented system is defined to be

$$\kappa_a(x,\mathbf{v}_a) = \frac{1}{2}\mathbf{v}_a^T\mathbf{M}_a(x)\mathbf{v}_a. \qquad (9)$$

We now define the augmented **desired** velocity field $V_a(x)$ given by

$$\mathbf{V}_a(x,t) = (V(x,t)\ ,\ V_f(x,t))\,, \qquad (10)$$

where $V(x,t)$ is the desired velocity which must be followed by the exercise machine control system (e.g. $V(x,t)$ may be given by Eq. (4) if $F = F_h$ and the functions $a(x)$ and $b(x)$ of the human force F_h in Eq. (3) are known). $V_f(x,t)$ is the desired velocity field of the fictitous flywheel. V_f must be chosen such that following conservation of energy condition is satisfied:

$$\overline{E} = \frac{1}{2}\mathbf{V}_a(x,t)^T\mathbf{M}_a(x)\mathbf{V}_a(x,t) \qquad (11)$$

for some constant $\overline{E}$, and for all x and t. By choosing $\overline{E}$ sufficiently large, $V_f(x,t)$ can be chosen to be the positive root of (11)

$$V_f(x,t) = \sqrt{1/M_f\{2\overline{E} - M(x)V^2(x,t)\}}. \qquad (12)$$

In order to define the velocity field control law, it is convenient to define the augmented momentum of the system

$$\mathbf{p}_a(x,t) = \mathbf{M}_a(x)\mathbf{v}_a, \qquad (13)$$

the augmented **desired** momentun

$$\mathbf{P}_a(x,t) = \mathbf{M}_a(x)\mathbf{V}_a(x,t) \qquad (14)$$

and the augmented system's *inverse dynamics*, evaluated at the augmentd desired velocity field

$$\mathbf{w}_a(x,t) = \mathbf{M}_a\dot{\mathbf{V}}_a(x,t) + \mathbf{C}_a(x,\dot{x})\mathbf{V}_a(x,t)\,, \qquad (15)$$

where $\dot{\mathbf{V}}_a(x,t) = \frac{\partial \mathbf{V}_a(x,t)}{\partial x}\dot{x} + \frac{\partial \mathbf{V}_a(x,t)}{\partial t}$. The velocity field control law is given by

$$\mathbf{T}_{a1} = \mathbf{Q}(x,\mathbf{v}_a,t)\,\mathbf{v}_a$$

where

$$\mathbf{Q} = \left[(\frac{1}{2\overline{E}}\mathbf{w}_a - \gamma\mathbf{p}_a)\mathbf{P}_a^T - \mathbf{P}_a(\frac{1}{2\overline{E}}\mathbf{w}_a - \gamma\mathbf{p}_a)^T\right] \qquad (16)$$
$$= \begin{pmatrix} 0 & q(x,\mathbf{v}_a,t) \\ -q(x,\mathbf{v}_a,t) & 0 \end{pmatrix}$$

is a skew-symmetric matrix, with

$$q(x,\mathbf{v}_a,t) = \frac{1}{2\overline{E}}\left[M(x)M_f(\dot{V}(x,t)V_f(x,t) - \dot{V}_f(x,t)V(x,t)) + M_fC(x,\dot{x})V(x,t)V_f(x,t)\right] + \gamma M(x)M_f(V(x,t)v_f - \dot{x}V_f(x,t))$$

and $\gamma > 0$. Notice that

$$\mathbf{v}_a^T\mathbf{T}_{a1} = \mathbf{v}_a^T\mathbf{Q}\mathbf{v}_a = 0\,.$$

Thus, the desired velocity field controller does not affect the overall energy level of the augmented system.

The properties of the passive velocity field controller are summarized in (Li and Horowitz, 1995; Li, 1995). Loosely speaking, in the absence of an external force F, if the mechanical system has some initial energy, the closed loop system will have a velocity $\dot{x}$ which asymptotically follows a multiple of the desired velocity field, i.e. $\dot{x}(t) \to \beta V(x,t)$, where the constant β depends on the energy level of the system.

3.2 *Overall Dynamic Damping Control*

In order to track the desired velocity filed V even when the external human force F is acting on the system, we must introduce in the coupling control action an additional control term that dissipates all the energy supplied by the user. This term is given by

$$\mathbf{T}_{a2} = -\mathbf{B}(x,t)\mathbf{v}_a, \qquad (17)$$

where $\mathbf{B}(x,t)$ must be positive definite and

$$\mathbf{B}(x,t)\mathbf{V}_a(x,t) = \begin{pmatrix} F_d(x,t) \\ 0 \end{pmatrix}. \qquad (18)$$

$F_d(x,t) = F_h(x,V(x,t))$ is the human force at the desired velocity. When (18) is satisfied, $\mathbf{T}_{a2}(x,t)$ exactly cancells out the modeled human force when the actual velocity $\mathbf{v}_a$ is such that $\mathbf{v}_a = \mathbf{V}_a$. If $F_d(x,t)V(x,t) > 0$, then $\mathbf{B}(x,t)$ can be chosen to be

$$\mathbf{B}(x,t) = \frac{F_d(x,t)}{2\overline{E}}\begin{pmatrix} M(x)V(x,t) & M_fV_f(x,t) \\ -M_fV_f(x,t) & M_fV(x,t) \end{pmatrix}. \qquad (19)$$

Combining the dynamics of the fictitious flywheel in (6), the coupling control term $\mathbf{T}_{a1}$ in (16) and the damping control term $\mathbf{T}_{a2}$ in (17), we obtain the following *dynamic damping* controller:

$$\mathbf{T}_a = \mathbf{T}_{a1} + \mathbf{T}_{a2} = \begin{pmatrix} T \\ M_f \dot{v}_f \end{pmatrix} = -\mathbf{R}(x, \mathbf{v}_a, t) \begin{pmatrix} \dot{x} \\ v_f \end{pmatrix} \quad (20)$$

where

$$\mathbf{R}(x, \mathbf{v}_a, t) = -\mathbf{Q}(x, \mathbf{v}_a, t) + \mathbf{B}(x, t), \quad (21)$$

$\mathbf{Q}(x, \mathbf{v}_a, t)$ satisfies Eq. (16) and $\mathbf{B}(x,t)$ satisfies Eq. (18).

The closed loop dynamics, consisting of the dynamics of the exercise machine in Eq. (2) and that of the dynamic damping controller in Eq. (20), is now given by

$$\mathbf{M}_a(x)\dot{\mathbf{v}}_a + \mathbf{C}_a(x, \mathbf{v}_a)\mathbf{v}_a = -\mathbf{R}(x, \mathbf{v}_a, t)\mathbf{v}_a + \mathbf{F}_a. \quad (22)$$

If the damping matrix $\mathbf{B}(x,t)$ is chosen according to Eq. (19), the matrix $\mathbf{R}(x, \mathbf{v}_a, t)$ in the resulting dynamic damping controller have skew symmetric off diagonal entries and is positive definite.

Let us define the augmented velocity error $\mathbf{e}_a := (e\ ,\ e_f)$ where $e(t) := \dot{x}(t) - V(x(t), t)$ and $e_f(t) := v_f(t) - V_f(x(t), t)$. Define also the Lyapunov function

$$W(x, \mathbf{e}_a) := \frac{1}{2}\mathbf{e}_a^T \mathbf{M}_a(x)\mathbf{e}_a \quad (23)$$

The properties of the exercise machine under the control of the dynamic damping controller in (20) are given in the following theorem.

Theorem 1. Suppose that the desired velocity field $V(x,t)$ and a Hill relation $F_h(x, \dot{x})$ satisfy $F_h(x, \dot{x})V(x,t) > 0$. Consider the dynamic damping controller in (20) with $F_d(x,t)$ in (18) given by $F_d(x,t) = F_h(x, V(x,t))$. The closed loop system given by (22) consisting of the exercise machine given by (2) and this dynamic damping controller, has the following properties:

(1) The closed loop system is passive with respect to the input/output pair $[F, \dot{x}]$ (i.e. Eq. (5) is staisfied for all $t \geq 0$ and any human force F).

(2) There exists an $\alpha > 0$ s.t. the Lyapunov function W in (23) satisfies

$$\dot{W}(t) \leq -\alpha W(t) + e(t)(F(t) - F_d(x(t), t)), \quad (24)$$

with $\alpha = 2 \min_{x,t} F_d(x,t)V(x,t)$.

(3) If $F(t) = F_h(x(t), \dot{x}(t))$, then $\mathbf{e}_a \to \mathbf{0}$ exponentially. Thus, $\dot{x}(t) \to V(x(t), t)$ as $t \to \infty$. The convergence rate is at least 0.5α, where α is given in item 2.

Proof: See (Li and Horowitz, 1997*b*; Li, 1995).

It is interesting to note that, the estimate of the convergence rate of $e \to 0$ is the minimum of the power generated by the human when moving along the desired velocity field.

3.3 *Controller Realization Using Mechanical Elements*

As detailed in (Li, 1995), the structure of the dynamic damping controller given by (20) has the advantage that it can be implemented using exclusively mechanically passive elements, without the need of a motor as in our experimental setup in Fig. 3. This can be accomplished by realizing the dynamics of the additional energy storage element in Eq. (6) as a mechanical spring instead of a fictitious flywheel. In this case $K = 1/M_f$ becomes the spring constant and $f = v_f$ the spring force. An implementation is shown in Fig. 4. It consists of the mechanical spring with spring constant K, and four variable dampers with damping coefficients $d_1(t), d_2(t), d_3(t)$ and $d_4(t)$.

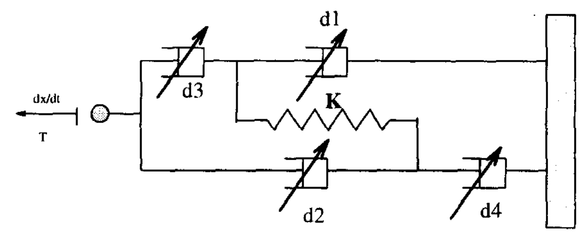

Fig. 4. A realization of the dynamic damping controller using passive mechanical elements

The dynamics of the realization shown in Fig. 4 is given by:

$$\begin{pmatrix} T \\ \frac{1}{K}\dot{f} \end{pmatrix} = \begin{pmatrix} -\left(\frac{d_1 d_3}{d_1 + d_3} + \frac{d_2 d_4}{d_2 + d_4}\right) & \left(\frac{d_3}{d_1 + d_3} - \frac{d_2}{d_2 + d_4}\right) \\ \left(-\frac{d_3}{d_1 + d_3} + \frac{d_2}{d_2 + d_4}\right) & -\left(\frac{1}{d_1 + d_3} + \frac{1}{d_2 + d_4}\right) \end{pmatrix} \begin{pmatrix} \dot{x} \\ f \end{pmatrix}$$

where T is the force output of the device concentric to the motion and f is the compressive force of the spring. The off diagonal components of this matrix are skew symmetric and the matrix itself is positive definite for positive $d_1, \cdots, d_4$, as is $\mathbf{R}(t)$ in (20) if the matrix $\mathbf{B}(x,t)$ is chosen according to Eq. (19). The reader is referred to (Li, 1995) for further details of the implementation.

4. STATIC DAMPING CONTROL

The dynamic dymping control presented in the previous section utilizes an energy storage element such as a spring or a flywheel. This is necessary to achieve both closed loop passivity and asymptotic tracking of an arbritary velocity field. The dynamic damping controller can be realized using a linear spring and four adjustable dampers, as was explained in section 3.3, and this avoids the use of potentially expensive active elements, such

DC motors, in the construction of the exercise machine. However, most exercise machines currently in the market are equipped with only one passive resistive element. In this section we present a control for exercise machines which utilizes only one static damping element and can be more readily implemented in current exercise machines. Unfortunately, this controller cannot achieve exact asymptotic tracking of arbitrary velocity fields. However, we have found in our experimental studies that this simplified control scheme is able to maintain a fairly small tracking error, particularly when the inertia $M(x)$ of the exercise system is not large. This control has been implemented on a Precor 855e recumbent bicycle, as well as the arm exercise machine which was described in section 2.1. As in the previous section we will develop the static damping controller assuming that the human force F in Eq. (2) is known. This assumption will be removed in section 5.3.

Consider again the exercise machine dynamics in Eq. (2), where F and T are respectively the generalized forces generated by the user's equivalent muscle and the controller. We now constrain the control to be a static damper of the form

$$T(x, \dot{x}, t) = -B_d(x,t)\dot{x}\,. \tag{25}$$

Assuming that F is known and is given by (3), the damping coefficient $B_d(x,t)$ can be chosen so that $T(x, V(x,t), t) = F_d(x,t) = F_h(x, V(x,t))$. If $V(x,t) > 0$ and $F_d(x,t) \geq 0$, $B_d(x,t)$ can be selected as

$$B_d(x,t) = \frac{F_d(x,t)}{V(x,t)} = \frac{a(x) - b(x)V(x,t)}{V(x,t)}. \tag{26}$$

However, this control only achieves asymptotic tracking of the desired velocity if $M(x)$, $a(x)$, $b(x)$ and $V(x,t)$ are constant, which is not the case on a typical exercise machine. To improve the control system's tracking performance, the following static damping control was proposed in (Shields, 1997; Shields and Horowitz, 1997)

$$\begin{aligned} T(x, \dot{x}, t) &= -B_{eff}(x, e, t)\dot{x} \\ B_{eff}(x, e, t) &= B_d(x,t) + m(e)e\,, \end{aligned} \tag{27}$$

where, $B_d(x, V(x,t), t)$ is given by (26), $e = \dot{x} - V(x,t)$ is the velocity tracking error and the function $m(e)$ is chosen as

$$m(e) = \frac{F_d(x,t)}{V(x,t)\,|e| + \alpha}, \tag{28}$$

where $\alpha > 0$.

The formula for $m(e)$ given in (28) provides a symmetric tracking correction force about the $e = 0$ equilibrium point, and assures that $B_{eff}(x,e,t) > 0$ as long as $F_d(x,t) \geq 0$ and the desired velocity field is constraint so that $V(x,t) < \frac{a(x)}{b(x)}$. In general only a fraction of the available braking power is used with the control (27) when $e > 0$, as there is no upper bound on the value of $m(e)$ when the actual velocity, $\dot{x}$, is too large. There may be instances were $m(e)$ should be modified to increase the braking power, however, a symmetric $m(e)$ is good for "comfort" reasons. Note that the passivity constraint $B_{eff}(t) > 0$ also guarantees that the control input will be non-positive, and that $\dot{x}$ is non-negative. This can be proven with mild assumptions by considering the solution to Eq. (2) with the input given by Eq. (27).

The error dynamics of the closed loop system with $m(e)$ chosen as in (28) can be written as:

$$\begin{aligned} M(x)\dot{e} + C(x)\dot{x}e &= -\left(\frac{a(x)}{V(x,t)} + m(e)\dot{x}\right)e \\ &+\tilde{F}(x,\dot{x},t) - M(x)\dot{V}(x,t) - C(x)\dot{x}V(x,t), \end{aligned} \tag{29}$$

where $\tilde{F} = F(x,\dot{x},t) - F_d(x,t)$. The following theorem establishes the boundedness of the tracking error.

Theorem 2. Assume that: (i) the human force $F(x,\dot{x},t) > 0$ is bounded, (ii) the desired velocity field is chosen such that $0 < V(x,t) < \frac{a(x)}{b(x)}$. Then:

(1) The closed loop system is passive with respect to the input/output pair $[F, \dot{x}]$,

(2) The velocity tracking error, e, associated with the error dynamics (29) satisfies,

$$|e| \leq \frac{\delta}{\lambda_{min}}, \tag{30}$$

where, $\delta = |d(t)|_\infty < \infty$, $\lambda_{min} = \min_t \lambda(t) > 0$, and $\lambda(t)$ and $d(t)$ are defined as,

$$\begin{aligned} \lambda(t) &:= \left(\frac{a(x)}{V(x,t)} + m(e)\dot{x}\right) \\ d(t) &:= \tilde{F}(x,\dot{x},t) - M(x)\dot{V}(x,t) - C(x)\dot{x}V(x,t). \end{aligned} \tag{31}$$

Proof: See (Shields, 1997; Shields and Horowitz, 1997).

Note that the user's isometric strength $a(x)$ parameter enters into the error dynamics (29) via the linear stabilizing term. Thus, the static damper provides improved tracking performance when the subject is stronger.

5. ADAPTIVE CONTROL

Both the dynamic damping control law given by (20) and the static damping control law given by (27) are functions of the human force $F_d(x,t)$. $F_d(x,t)$ is the force that the user would exert at the position x if the velocity of motion is as desired, i.e. $\dot{x} = V(x,t)$. Given and *arbitrary* desired velocity profile $V(x,t)$, the only unknown factor in Eqs. (20) and (27) is $F_d(x,t)$, since it depends on the unknown biomechanics of the user. If the affine force-velocity model for the user's biomechanics given by Eq. (3) is an accurate model of the user's force, then $F_d(x,t)$ should be given by

$$F_d(x,t) = F_h(x, V(x,t)) = a(x) - b(x)V(x,t). \tag{32}$$

5.1 *Linear parametrization of the unknown functions $a(x)$ and $b(x)$*

In order to estimate the unknown functions $a(x)$ and $b(x)$ in the linearized Hill model (3), we assume that these functions can be parameterized via an integral representation.

Let $k : \mathcal{G} \times \mathcal{G} \to \Re$ be sufficiently smooth, symmetric and has a finite eigenfunction expansion, i.e. there exists $\psi_1(\cdot), \ldots, \psi_N(\cdot) : \mathcal{G} \to \Re$, s.t.

$$\int_{\mathcal{G}} k(x,\sigma)\psi_i(\sigma)d\sigma = \zeta_i \psi_i(x), \quad \int_{\mathcal{G}} \mathbf{\Psi}^T(\sigma)\mathbf{\Psi}(\sigma)d\sigma = I_N$$
$$k(x,\sigma) = \mathbf{\Psi}(x)\mathbf{Z}\mathbf{\Psi}^T(\sigma), \quad \mathbf{Z} = \mathrm{diag}(\zeta_1, \cdots, \zeta_N),$$

where $\zeta_i > 0$ and $\mathbf{\Psi}(\cdot) = [\psi_1(\cdot), \cdots, \psi_N(\cdot)]$.

Assumption 1. For a given neural input, the muscle force is given by the Hill model in (3), with $a(x) \in [\underline{a}(x), \infty)$ and $b(x) \in [\underline{b}(x), \bar{b}(x)]$, where $\underline{a}(x)$, $\underline{b}(x)$, $\bar{b}(x) > 0$.

Assume that $a, b : \mathcal{G} \to \Re^+$ can be represented by an integral equation of the first kind (Messner et al., *1991),*

$$\begin{pmatrix} a(x) \\ b(x) \end{pmatrix} = \int_{\mathcal{G}} k(x,\sigma) \begin{pmatrix} c_a(\sigma) \\ c_b(\sigma) \end{pmatrix} d\sigma \tag{33}$$

where $c_a(\cdot)$, and $c_b(\cdot)$ are squared integrable. Moreover, the vector function $\mathbf{c} : \mathcal{G} \to \Re^2$, $\mathbf{c}(x) = (c_a(x), c_b(x))^T$, lies in a convex set $\mathcal{P}_c$ so that $a(x) \in [\underline{a}(x), \infty)$ and $b(x) \in [\underline{b}(x), \bar{b}(x)]$.

The functions $k(\cdot,\cdot)$ and the $c_a(\cdot)$ and $c_b(\cdot)$ are referred to as the kernel and the influence functions respectively. The integral representation has been found to be useful in functional adaptive control (Messner *et al.*, 1991).

Defining the functional vector regressors for each $\sigma \in \mathcal{G}$,

$$\phi^c(t,\sigma) = k(x(t),\sigma)\big(1, \ -\dot{x}(t)\big)^T, \tag{34}$$
$$\phi_d^c(t,\sigma) = k(x(t),\sigma)\big(1, \ -V(x,t)\big)^T$$

and the corresponding vector regressors and parameter vectors

$$\phi(t) = \int_{\mathcal{G}} \phi^c(t,\sigma)\mathbf{\Psi}(\sigma)d\sigma, \qquad \phi_d(t) = \int_{\mathcal{G}} \phi_d^c(t,\sigma)\mathbf{\Psi}(\sigma)d\sigma,$$
$$\mathbf{\Theta} = \int_{\mathcal{G}} \left(\mathbf{\Psi}^T(\sigma) \ \mathbf{\Psi}^T(\sigma)\right)^T \mathbf{c}(\sigma)d\sigma, \tag{35}$$

we can obtain parameterizations for the human generalized force F_h in (3) and $F_d(x,t)$ in (32)

$$F_h(x,\dot{x}) = \int_{\mathcal{G}} \phi^c(x,\dot{x},\sigma)^T \mathbf{c}(\sigma)d\sigma = \phi(t)^T\mathbf{\Theta}, \tag{36}$$
$$F_d(x,t) = \int_{\mathcal{G}} \phi_d^c(t,\sigma)^T \mathbf{c}(\sigma)d\sigma = \phi_d(t)^T\mathbf{\Theta},$$

where $\mathbf{c}(\cdot) = (c_a(\cdot), c_b(\cdot))^T$ and $\mathbf{\Theta} = (\theta_a, \theta_b)^T$.

The integral representation (36) is used in the actual implementation of the estimation algorithms. However, the equivalent vector parametrization (36) is more useful in the analysis. We use only the more familiar vector parametrization (36) in the remainder of the paper.

5.2 *Force Observer*

The muscle force F cannot generally be measured even with a force sensor unless invasive techniques are used. For the estimation of the unknown parameter vector $\mathbf{\Theta}$ in (36), we shall utilize a force observer to obtain $\mathcal{F}(t)$, which is the stable filtered output of $F(t)$, using only position and velocity measurements,

$$\mathcal{F}(t) = \frac{\lambda}{\lambda + s} F(t), \qquad \lambda > 0. \tag{37}$$

Since T, $M(x)$ and $C(x)$ are known, it can be verified from (2) that $\mathcal{F}(t)$ can be computed by

$$\mathcal{F} = \lambda M(x)\dot{x} - \frac{\lambda}{\lambda + s} r(t) \ , \tag{38}$$
$$r(t) := T(t) + C(x(t))\dot{x}(t) + \lambda M(x(t))\dot{x}(t).$$

The filtered force $\mathcal{F}(t)$ is related to the biomechanics parameters $\mathbf{\Theta}$ by

$$\mathcal{F}(t) = \rho(t)^T\mathbf{\Theta}, \quad \rho(t) := \frac{\lambda}{s + \lambda}\phi(t). \tag{39}$$

5.3 *Adaptive Damping Control*

Using the certainty equivalence approach, we now replace $F_d(x,t)$ in either the dynamic damping control law in Eq. (20) or the static damping law in Eq. (27) (32) by its estimate $\hat{F}_d(x,t)$,

$$\hat{F}_d(x,t) = \hat{a}(x,t) - \hat{b}(x,t)V(x,t) = \phi_d(t)^T\hat{\mathbf{\Theta}}(t) \tag{40}$$

and $\phi_d(t)$ is defined in (35).

The parameter estimate vector $\hat{\mathbf{\Theta}}$ is updated using the following Parameter Adaptation Algorithm (PAA),

$$\dot{\hat{\mathbf{\Theta}}}(t) = -\,\mathrm{Proj}_{\hat{\mathbf{\Theta}}}\left[\mathbf{P}(t)\hat{\mathbf{\Theta}}(t) - \mathbf{d}(t) + \phi(t)e(t)\right], \tag{41}$$
$$\dot{\mathbf{P}}(t) = -\lambda\mathbf{P}(t) + \rho(t)\rho^T(t), \qquad \mathbf{P}(0) = \mathbf{0}, \tag{42}$$
$$\dot{\mathbf{d}}(t) = -\lambda\mathbf{d}(t) + \rho(t)\mathcal{F}(t), \qquad \mathbf{d}(0) = \mathbf{0} \tag{43}$$
$$e(t) = \dot{x}(t) - V(x,t)\ , \tag{44}$$

where $\phi(t)$ is defined in (35), $\rho(t)$ is defined in (39), $\mathcal{F}(t)$ is given by (38) and $\text{Proj}_{\hat{\Theta}}$ is the projection operator that makes sure that $\hat{\Theta}$ remains in the proper set $\mathcal{P}_c$ specified in Assumption 1 so that Hill parameter estimates $\hat{a}(t,x)$, $\hat{b}(t,x)$ satisfy $\hat{a}(t,x) \geq \underline{a}(x)$, $\hat{b}(t,x) \in [\underline{b}(x), \overline{b}(x)]$.

Notice that the PAA in (41) is driven by two adaptation error signals: the "input" error signal $\mathbf{P}\hat{\Theta} - \mathbf{d}$, and the "output" error signal $e = \dot{x} - V$. The combination of output and input error signals in a PAA was first suggested by (Slotine and Li, 1989) for robot manipulator tracking control problems.

The update law for the gain matrix $\mathbf{P}$ in (42) and that definition of the error $\mathbf{P}\hat{\Theta} - \mathbf{d}$ is based on the work of (Kreisselmeier, 1989). To understand the meaning of the error term $\mathbf{P}\hat{\Theta} - \mathbf{d}$, multiply both sides of (42) by Θ and observe that $\mathbf{P}(t)\Theta$ and $\mathbf{d}(t)$ satisfy the same ODE and have the same initial condition. Therefore, by the uniqueness of the solution of ODEs, $\mathbf{P}(t)\Theta = \mathbf{d}(t)$. Hence, the PAA (41) can be re-written as

$$\dot{\hat{\Theta}}(t) = -\text{Proj}_{\hat{\Theta}}\left[\mathbf{P}(t)\tilde{\Theta}(t) + \phi(t)e(t)\right], \qquad (45)$$

where $\tilde{\Theta}(t) = \hat{\Theta}(t) - \Theta$.

Figure 5 shows a block diagram of the overall adaptive damping control system. Although the figure shows a static certainty equivalence damping control block in the system, this block could be the dynamic certainty equivalence damping control.

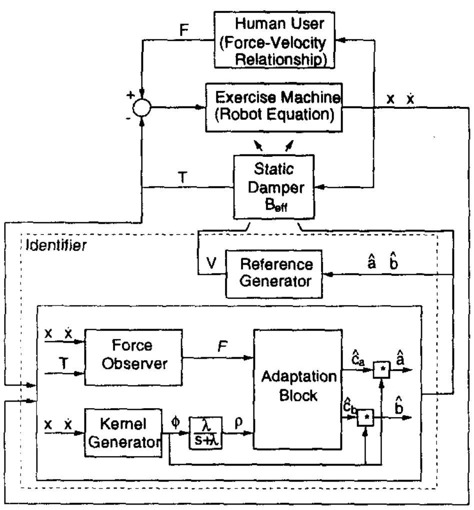

Fig. 5. Control architecture block diagram

The following theorem shows that when the certainty equivalence dynamic damping control law is utilized, the adaptive controller is capable of tracking an arbitrary desired velocity field, while interacting passively with the human user. A weaker bounded tracking error result, similar to Theorem 2, can be obtained if the certainty equivalence static damping control law is utilized (Shields, 1997; Shields and Horowitz, 1997).

Theorem 3. Suppose that the desired velocity profile $V(x,t)$ satisfies $0 < V(x,t) < \frac{\underline{a}(x)}{\overline{b}(x)}$ where $\underline{a}(x), \overline{b}(x)$ are the bounds on the Hill parameters in Assumption 1.

The certainty equivalence controller consisting of the dynamic damping controller (20) with $F_d(x,t)$ replaced by $\hat{F}_d(x,t)$ in Eq. (40), and the identifier (41)-(44) has the following properties:

(1) The closed loop system is passive w.r.t. the input output pair $[F, \dot{x}]$.

(2) Let $e = \dot{x} - V(x,t)$, $e_f = v_f - V_f(x,t)$, and $\mathbf{e}_a = (e, e_f)^T$. Suppose that the user's force $F(t)$ is given by a Hill relation that satisfies Assumption 1 so that

$$F(t) = F_h(x(t), \dot{x}(t)) := a(x) - b(x)\dot{x}.$$

Then, $\mathbf{e}_a \to \mathbf{0}$. Thus, the desired velocity field V is asymptotically followed.

Proof: See (Li and Horowitz, 1997*b*; Li, 1995).

Although the results in theorem 3 can be achieved using a simpler PAA, like a gradient PAA, the filtered gain matrix update law (42) with the input error term $\mathbf{P}\hat{\Theta} - \mathbf{d} = \mathbf{P}\tilde{\Theta}$ enhances the convergence of the parameter error to zero and is needed in the development of the self-optimizing control strategy, which is breifly discussed in the next section.

6. SELF-OPTIMIZING CONTROL STRATEGY

The dynamic damping adaptive control in the previous section is capable of following an arbitrary desired velocity field. However, in order for the control system to achieve an optimal exercise regime, it is necessary that the desired velocity field be given by $V^*(x)$ in (4), which depends on the unknown Hill surface functions $a(x)$ and $b(x)$. Since $a(x)$ and $b(x)$ are unknown, we can therefore define the optimal desired velocity field *estimate*

$$\hat{V}^*(x,t) \doteq \frac{\hat{a}(x,t)}{2\hat{b}(x,t)}, \qquad (46)$$

which is based on the parameter estimate $\hat{\Theta}$. However, an optimal exercise regime is only achieved if $\hat{V}^*(x,t) \to V^*(x)$, which is guaranteed if $\hat{\Theta} \to \Theta$. Unfortunately, the optimal velocity field estimate $\hat{V}^*(x,t)$ may not have enough excitation to guarantee that $\hat{\Theta} \to \Theta$ or $\hat{V}^*(x,t) \to V^*(x)$.

A self-optimizing control strategy was introduced in (Li and Horowitz, 1994; Li, 1995) to solve the this problem. In this strategy a reference velocity field generator is used to provide the exercise control system with different desired velocity fields. The reference generator will either provide the optimal velocity field estimate

$\hat{V}^*(x,t)$ in (46) or a training velocity field $V^{tr}(x,t)$, which provides sufficient excitation so that the true Hill surface functions $a(x)$ and $b(x)$ can be identified. Since the linearized Hill surface model in Eq. (3) is affine in the velocity $\dot{x}$ at each position x, the training velocity profile must at least visit sufficiently often two different velocities at each position x to provide persistence of excitation, so as to properly identify the Hill surface functions $a(x)$ and $b(x)$. Thus, the training velocity field $V^{tr}(x,t)$ is specified to be the time varying weighted average between two constant velocity fields V_{high} and V_{low}.

A finite state machine excitation supervisor switches between these two desired velocity fields, based on an error signal which provides information on the accuracy of the estimated optimal velocity field. Switching is performed in a manner that, when the optimal velocity field estimate is inaccurate, the training velocity field is more frequently chosen. On the other hand, the optimal velocity field estimate is more frequently chosen when the finite state machine supervisor determines that it is accurate. A block diagram of the overall self-optimizing control system is depicted in Fig. 6.

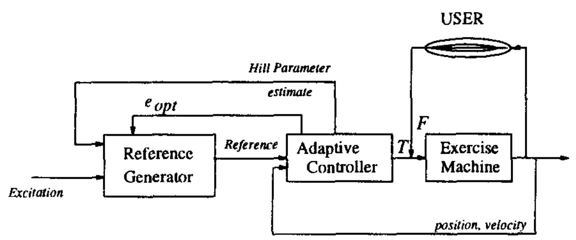

Fig. 6. Self-optimizing control system

A state transition between the training or optimal velocity fields can be generated by either a deterministic or stochastic criterion. As shown in (Li and Horowitz, 1997*b*; Li, 1995), under the assumption that the Hill surface is time invariant, when the deterministic excitation supervisor is used, only a finite number of training phases occur after which the user exercises at the optimal velocity profile. Similarly, if stochastic excitation supervisor is used, the probability that a training phase occurs vanishes asymptotically with time, and it is almost certain that the user performs the optimal exercise.

In an actual implementation, the assumption that the user's force is governed by a time invariant Hill surface is not strictly true due to the variability of the effort level, fatigue and other factors. However, by setting the decision parameters in the excitation supervisor appropriately, the excitation supervisor can be made insensitive to small variations in the Hill surface, while still responding to sufficiently large variations, and transitioning into the training state, so as to learn the new Hill surface. Interested readers are referred to (Li and Horowitz, 1997*b*; Li, 1995) for details.

The self-optimizing control strategy was implemented on the experimental setup (Fig. 3) described in section 2.1. The subject was instructed to exercise at a constant effort level as far as he could discern. The certainty equivalent dynamic damping control law was utilized in these tests. The stochastic excitation supervisor was used to determine the transitions between training and opti-poteric desired velocity fields. Experimental results are shown in Figs. 7 and 8. As shown in Fig. 7, the

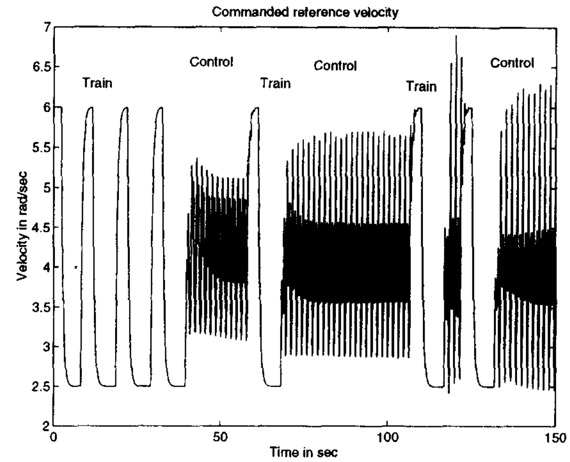

Fig. 7. Reference velocity profile: $V(x(t),t)$

desired velocity profile $V(x(t),t)$ consists of the training velocity profiles which are alternate constant velocities at $V_{high} = 6$ rad/s and $V_{low} = 2.5$ rad/s, and the estimated optimal velocity profiles. The figure shows how the stochastic excitation supervisor switches between the training and optimal exercising regimes. From Fig. 8, where the actual velocity and the desired velocity are plotted, we see that the certainty equivalent dynamic damping control law was able to cause the subject to execute the desired exercise profile during both the the training and optimal exercising regimes.

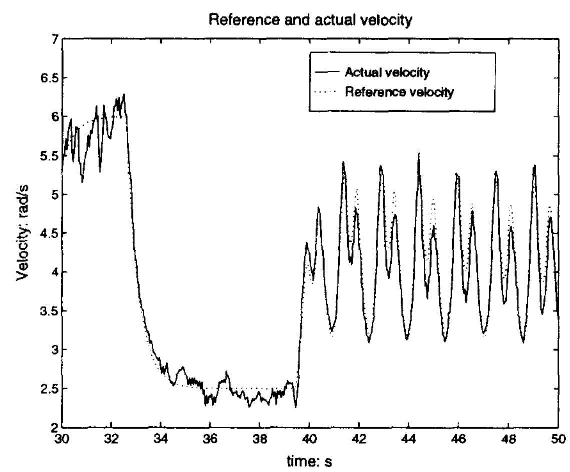

Fig. 8. Reference and actual velocity versus time

7. CLINICAL STUDIES

In this section the results of clinical studies using the arm cranking exercise described in section 2.1 and the certainty equivalence static damping control law are reported. The purpose of these studies is to verify the effectiveness of the linearized model in Eq. (3) in describing the strength characteristics of human subjects and to see if it can be used to successfully implement the optipoteric exercise, both without fatigue, and in the presence of fatigue. The benchmark for comparison to the optipoteric exercise were isokinetic (constant velocity) exercises at three different velocity levels. Two exercise protocols were used. The first was a shorter protocol done at a low effort level. Its purpose was to verify the assumptions made about using the strength surface as a method of optimizing the flow of energy from the exerciser to the machine. The second protocol was designed to test if the optimality of the optipoteric exercise regime held even when the user is fatiguing. This second test was particularly critical in light of the results in (Beelen and Sargeant, 1991; McCartney *et al.*, 1983), where is was found that the fatigue rate (as measured by the decrease in power over a certain interval of time) for low velocity isokinetic exercise was less then the fatigue rate for high velocity isokinetic exercise.

Both the low effort protocol and maximum effort protocol started with an excitation phase, where the desired velocity was switched between low and high velocities. This was necessary to satisfy the persistence of excitation requirements described in section 6. During this excitation phase the subjects force-velocity parameter functions, $a(x,\cdot)$ and $b(x,\cdot)$, were estimated. These parameters are used to implement the certainty equivalence static damping control law, (27), and for specifying the optimal velocity field $V^*(x)$ in (4) later in the protocol. For the low effort protocol, the excitation phase (30 seconds) was followed by a randomized sequence of three isokinetic treatments (20 seconds each at 2.0, 4.5, and 7.0 rad./sec.) and the optipoteric treatment (20 seconds). The randomization was done so that if any fatigue occurred during the protocol, its effect would not bias the later treatments. Six normal male college students volunteered for this portion of the study. For the maximum effort protocol, the excitation phase (55 seconds) was followed by one of the isokinetic (195 seconds at 2.0, 4.5, and 7.0 rad./sec.) or optipoteric (195 seconds) treatments. The treatment lasted for enough time so that a significant drop in the power level of the exercise was produced. At the end of the treatment a final excitation phase (30 seconds) was implemented so that the force-velocity parameters of the subject could be determined after the fatigue episode. Only one protocol per day was performed to allow the subjects adequate recovery time between the four experiments. Four normal college students volunteered for this protocol.

For the maximum effort protocol, the neural activation was likely to be constant within the protocol and across different protocols for each of the treatments (Pertuzon and Bouisset, 1973). For the low effort protocol this was not the case, as a subjects impression of their effort level is not reliable (Merton, 1954). As a consequence, it was necessary to verify that the effort level during this protocol was constant, otherwise the difference in power output of each treatment could be due to factors other then the velocity difference. To check the effort level, the processed electromyogram (rectified and filtered) signal from the muscles being exercised was displayed on an oscilloscope. The subjects could then monitor their neural activation by looking at the scope trace and make any adjustments necessary if they observed a change in the level of the trace. Although the strength surface is studied for a constant effort level at each velocity, the scheme proposed in this paper is not limited to this case. The optimization can be based on whatever exercise strategy the user decides to pursue for the different desired velocities.

To evaluate the effectiveness of the isokinetic and optipoteric treatments in terms of maximizing the rate of energy dissipated by the user, a filtered power signal was used. Since the true instantaneous power, $P = F \cdot \dot{x}$, is not available because F is not measurable, a "filtered" power variable was computed by

$$\mathcal{P} = \mathcal{F} \cdot \dot{x}, \qquad (47)$$

where $\mathcal{F} = \frac{\lambda}{s+\lambda} F$ is the output of the force observer, given by Eq. (37). If λ is chosen large enough, the bandwidth of the filter for $\mathcal{F}$ will be large and $\mathcal{F}$ will closely approximate F.

To obtain average values of $\mathcal{F}$ and $\mathcal{P}$ over a cycle of the exercise motion, the data was processed through an FIR boxcar filter. The coefficients of the filter were set to give unity gain and the tail of the filter was longer then the amount of time for one period of the low velocity motion. The averaged filtered torque and averaged filtered power signals will be denoted by $\mathcal{F}_{ave}$ and $\mathcal{P}_{ave}$ respectively. The total amount of work performed during the maximum effort protocol was calculated using the integral of the averaged filtered power signal,

$$Work = \int_{t_1}^{t_2} \mathcal{P}_{ave}(\tau) d\tau. \qquad (48)$$

where t_1 and t_2 are the times just after the initial excitation phase and just before the final excitation phase.

7.1 *Results*

Table 1 summarizes the results of the low effort protocol for the arm machine. Values of filtered power are means over the duration of the treatments indicated. Values of estimated optimal velocity are means over the duration of the optipoteric treatment. Using upper-tailed paired t-tests the sample mean $\overline{\mathcal{P}}$ for the optipoteric treatment was found to be significantly greater then the sample mean $\overline{\mathcal{P}}$ for the low, medium, and high velocity isokinetic treatments at confidence levels of 99%, 95%, and 95% respectively. This establishes that the identifier does determine the velocity that corresponds to the peak in power according to the torque-velocity relationship of the subject, and, furthermore, that the static damping controller is successful in stabilizing the motion close enough to the estimated optimal velocity so that the power of the optipoteric treatment is optimal relative to the isokinetic treatments. Figure 9 illustrates the superb tracking ability of the static damping control law, described in section 4, in tracking the optipoteric desired velocity field, $V^*(x)$, for the arm exercise machine. The tracking error in this case is not significantly worse then that achieved by the dynamic damper control law described in section 3.

Subject:	1	2	3	4	5	6	Mean±Std.
Opti.:	72.8	50.4	69.4	58.3	68.8	52.9	62.1±9.47
Low Iso. :	34.9	36.6	44.7	41.2	51.7	35.7	40.8±6.52
Med. Iso.:	56.4	44.5	64.5	49.5	74.6	37.1	54.4±13.7
High Iso.:	32.4	34.7	19.8	59.4	62.9	17.1	37.7±19.4
Mean. V^*:	5.79	4.57	4.07	5.64	4.62	4.01	4.78±0.77

Table 1. Arm machine mean filtered power $\mathcal{P}$ in Watts, and mean estimated optimal velocity V^* in rad./sec., for the different treatments of the low effort protocol.

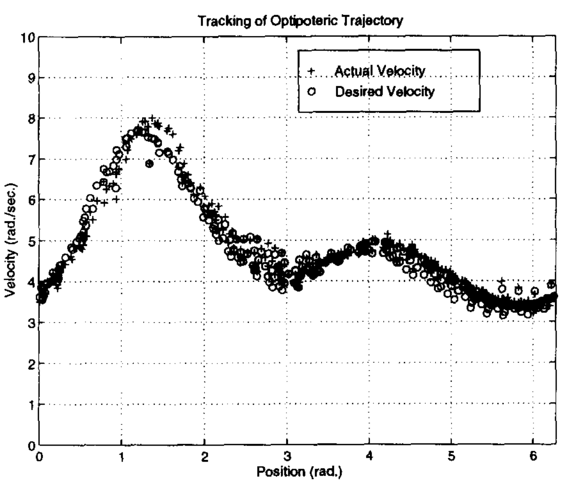

Fig. 9. Tracking performance of the static damper for the arm exercise machine during the optipoteric phase.

Figure 10 shows the time dependence of $\mathcal{P}_{ave}$ during the intended exercise portion of the maximum effort protocol. The total amount of work during the intended exercise portion of the protocol was calculated for each treatment using Eq. (48). The results are listed in table 2. The optipoteric treatment gave statistically significant greater power then all three of the isokinetic treatments at a confidence level of 95%. Thus, the long term optimality of the optipoteric profile has been shown and the fear that the larger amount of fatigue at a high velocity optipoteric treatment may cause long term suboptimality of the treatment was shown to be unfounded for this exercise regime.

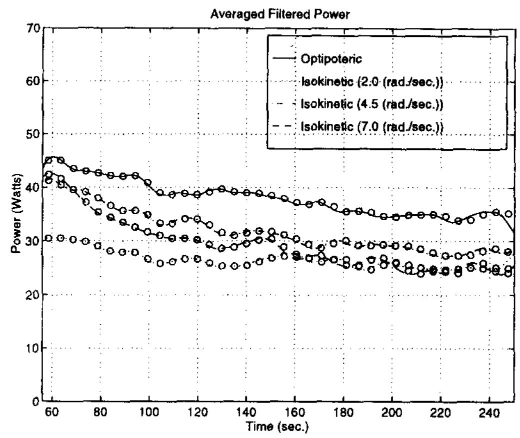

Fig. 10. Averaged filtered power during the intended exercise portion of the maximum effort protocol for the arm exercise machine. Each curve represents the average of the four subjects.

Subject:	1	2	3	4	Mean±Std.
Opti. :	10.0	7.70	7.06	4.79	7.41±2.18
Low Iso.:	6.10	5.75	4.83	3.87	5.14±1.00
Med. Iso.:	8.24	7.25	5.21	4.12	6.20±1.87
High Iso.:	9.26	5.02	4.89	3.57	5.68±2.47
Pre-Fat. V^*:	5.19	4.88	4.59	4.65	4.83±0.27
Post-Fat. V^*:	4.66	4.07	4.02	4.01	4.19±0.31

Table 2. Work in kJoules performed on the arm exercise machine during the intended exercise portion of the fatiguing protocol. Pre- and post-fatigue estimated optimal velocities, V^*, are in units of rad./sec..

The results presented in this section also demonstrate that the opti-poteric velocity is highly variable, validating the need for an adaptive scheme, such as the one proposed in this paper, to optimize the external power output. The low effort protocol has shown that the optimal velocity varies from subject to subject. The pre- and post-fatigue optimal velocities for the maximum effort protocol have shown that the optimal velocity varies with fatigue. By comparing the optimal velocities for the low and maximum effort protocols the optimal velocity has been shown to vary with effort level. The position dependence of the optimal velocity has also been demonstrated.

8. CONCLUSION

In this paper we discussed the control of self-optimizing exercise machines. Based upon known human biomechanical characteristics, a linearized generalized force-velocity relationship was postulated, which models the response of muscular generalized forces acting on the exercise machine in the neighborhood of an operating velocity trajectory. A new opti-poteric exercise strategy was describe, in which the instantaneous power produced by the human user is optimized, and the exercise machine control objective was define in terms of tracking the unique optimal velocity field at which the user exercises at the maximum power level. Certainty equivalence adaptive dynamic and static damping control laws were formulated which enforce that the closed loop system always interacts passively with the human user and track an arbitrary desired velocity field, even under the action of the user's external forces. The dynamic damping control law asymptotically achieves a zero tracking error, while the static damping control law achieves ultimate boundedness of the tracking error. A self-optimizing control strategy was briefly described that allows the control system to identify the biomechanics of the user and subsequently determine the optimal velocity field to be tracked by the control system. Experimental results were presented that verify many of the assumptions postulated in the paper. In particular, clinical studies performed on a arm-cranking exercise machine were presented that verify the feasibility of the proposed control scheme both with and without a fatigue episode. It was shown that the proposed control system is capable of identifying and tracking the opti-poteric velocity field which maximizes the instantaneous power produced by the user. Moreover, the opti-poteric exercise workout produced the maximum user overall energy consumption, relative to benchmark isokinetic exercises, even if significant fatigue is occurring during the workout.

9. REFERENCES

Beelen, A. and A. J. Sargeant (1991). Effect of fatigue on maximal power output at different contraction velocities in humans. *Journal of Applied Physiology* **71**(6), 2332–2337.

Hill, A.V. (1938). The heat of shortening and the dynamic constants of muscle. *Proceedings of the Royal Society B* pp. 36–195.

Kreisselmeier, G. (1989). Indirect adaptive control with a self-excitation capability. *IEEE Transaction on Automatic Control* **34**(5), 524–528.

Lehman, S.L. and B.M. Calhoun (1990). An identified model for human wrist movements. *Experimental Brain Research* **81**, 199–208.

Li, Perry Y. (1995). Self-Optimizing Control and Passive Velocity Field Control of Intelligent Machines. PhD thesis. University of California at Berkeley.

Li, Perry Y. and Roberto Horowitz (1994). On the optimizing-adaptive control problem. In: *Proceedings of the 1994 American Control Conference, Baltimore MD.* pp. 2170–2174.

Li, Perry Y. and Roberto Horowitz (1995). Passive velocity field control of mechanical manipulators. In: *Proceedings of the 1995 IEEE International Conference on Robotics and Automation, Nagoya, Japan.*

Li, Perry Y. and Roberto Horowitz (1997*a*). Control of smart exercise machines, part I: Problem formulation and non-adaptive control. *IEEE/ASME Transactions on Mechatronics*, **2**(4).

Li, Perry Y. and Roberto Horowitz (1997*b*). Control of smart exercise machines, part II: Self-optimizing control. *IEEE/ASME Transactions on Mechatronics*, **2**(4).

Lieber, Richard L. (1992). *Skeletal Muscle Structure and Function, Implications for Rehabilitation and Sports Medicine.* Williams and Wilkins.

McCartney, N., J. F. Heigenhauser and N. L. Jones (1983). Power output and fatigue of human muscle in maximal cycling exercise. *Journal of Applied Physiology* **55**(1), 218–224.

McMahon, T.A. (1984). *Muscles, Reflexes, and Locomotion.* Princeton University Press. Princeton.

Merton, P. A. (1954). Voluntary strength and fatigue. *Journal of Physiology* **123**, 553–564.

Messner, W., R. Horowitz, W. Kao and M. Boals (1991). A new adaptive learning rule. *IEEE Trans. Automatic Control* **36**(2), 188–197.

Pertuzon, E. and S. Bouisset (1973). Instantaneous force-velocity relationship in human muscle. In: *Biomechanics III.* Vol. 8. University Park Press. Rome. pp. 230–234.

Shields, J. (1997). Control of Exercise Machines: Theory and Experiments. PhD thesis. University of California at Berkeley. Dept. of Mechanical Engineering.

Shields, Joel and Roberto Horowitz (1997). Controller design and experimental verification of a self-optimizing motion controller for smart exercise machines. In: *Proceeding of the American Control Conference (ACC).*

Slotine, J-J. and W-P. Li (1991). *Applied Nonlinear Control.* Prentice Hall.

Slotine, J.J. and W.P. Li (1989). Composite adaptive control of robot manipulators. *Automatica* **25**(4), 509–519.

Zatsiorsky, V. and V. Seluyanov (1983). Estimation of the mass and inertia characteristics of the human body by means of the best predictive regression equations. In: *Proceedings of the 9th International Congress of Biomechanics.* Waterloo, Canada.

ZODIAC, The (1996). *Theory of Robot Control.* Springer-Verlag. Carlos Canudas-de-Wit, Bruno Siciliano and Georges Bastin (Edit.).