A Multi-rate Nonlinear State Estimator for Hard Disk Drives

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Abstract

In this paper, we propose a new multi-rate state estimator for seeking control of hard disk drives, which has a proportional, integral, and discontinuous error feedback structure to improve robustness. Conventional state estimators do not have enough accuracy under the existence of external disturbances and model uncertainty. With the proposed estimator, the seeking control system of hard disk drives can have better robustness. Our main focus is on the discontinuous action, which alleviates the estimation error caused by, for example, actuator gain variations. Integral action is secondarily introduced not only to cancel a quasi-constant disturbance and uncertainty, but also to reduce the chattering motion caused by the discontinuous feedback. Simulations and experiments have been carried out to validate our proposed multi-rate state estimator. Finally, an experimental example of seeking control for a single stage actuated hard disk drive is demonstrated.

1. Introduction

Since the utilization of multi-rate feedback control for hard disk drives (HDDs) was proposed[1], many authors have discussed this issue and it is widely used in order to achieve smoother control input and higher control bandwidth under lower sampling frequency. Among them, some have concentrated on developing a multi-rate state estimator[2] that yields smoother control input and higher control bandwidth under lower sampling frequency. In this paper, we combine the basic idea of sliding mode observer and integral observer, each of which are respectively introduced by Hara[6] and Beale[9], and put them into a multi-rate state estimator framework, as originally proposed by Hara[2], in order to reduce the problem of chattering motion. Moreover, we consider the following system:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + w(x,u,t) \\
y(t) &= Cx(t)
\end{align*}
\]

where \( x \in \mathbb{R}^n \), \( y \in \mathbb{R}^p \), \( u \in \mathbb{R}^m \), \( A \in \mathbb{R}^{n \times n} \), \( B \in \mathbb{R}^{n \times m} \), \( C \in \mathbb{R}^{p \times n} \), and we assume that the pair \((A,B)\) is controllable, the pair \((A,C)\) is observable, \( B \) and \( C \) are both full rank. Time delay is ignored for simplicity. The equivalent input multi-rate system can be described by:

\[
\begin{align*}
\dot{x}(k,i+1) &= \Phi_n x(k,i) + \Gamma_n u(k,i) + w(x,u,k) \\
y(k,i) &= Cx(k,i)
\end{align*}
\]

of which \( \Phi_n \in \mathbb{R}^{n \times n} \) and \( \Gamma_n \in \mathbb{R}^{n \times m} \) can be obtained as follows:

\[
\Phi_n = e^{AT_n}, \Gamma_n = \int_0^t e^{Au} \, dt
\]

where \( T_n = \) a control input updating time that can be described by \( T_n = T_r / r \), of which \( T_r \) is a measurement sampling time, and \( r \) is a multi-rate ratio. \((k,i)\) denotes the time \( t = kT_r + iT_n \) for \( i = 0,1,...,r-1 \). \( w \in \mathbb{R}^p \) represents any uncertainty and nonlinearity belonging to the range space of some matrix, \( \Theta \in \mathbb{R}^{n \times q} \), with restriction, \( q \leq p \), and it can be thought of as comprising the average and fluctuating behaviors. Thus \( w \) can be described as follows:

\[
w(x,u,k) = \Theta \{ h + \delta h(x,u,k) \}
\]

where \( h \in \mathbb{R}^q \) is a quasi-constant vector that represents the average uncertainty behavior, and \( \delta h \in \mathbb{R}^q \) represents the fluctuating behavior. In addition, we assume:

\[
\| \delta h(x,u,k) \| < h^*(x,u,k)
\]

where \( h^* \in \mathbb{R}^q \) is a known scalar function. We also assume that \( \delta h \) is slowly varying. Practically, by slowly varying, we mean its frequency is less than, at least, a quarter of possible update frequency of variable feedback structure.

Throughout this paper, \( \rho(\cdot) \) denotes the spectral radius, \( \| \cdot \| \) denotes the Euclidian norm for vectors and
the spectral norm for matrices, and \( \lambda_{\text{min}}(\bullet) \) denotes the smallest eigenvalue.

2. Estimator with integral feedback

The multi-rate state estimator proposed by Hara can be described as follows:

\[
\begin{align*}
\bar{x}(k,i) &= \left\{ \Phi^r(k-1,r-1) + \Gamma \right\} u(k-1,r-1) \cdots (i = 0) \\
\hat{x}(k,i) &= \bar{x}(k,i) + L_i(y(k,0) - C\bar{x}(k,0)) \\
\dot{\hat{x}}(k,i) &= \hat{x}(k,i) + L_i\{y(k,0) - Cx(k,0)\} + \Theta \dot{\hat{x}}(k,0) \\
\dot{\hat{z}}(k,0) &= \bar{x}(k,0) + K\{y(k,0) - Cx(k,0)\}
\end{align*}
\]

(2.1)

(2.2)

(2.3)

where \( \bar{x} \) is a prediction estimate of \( x \), \( \dot{\hat{x}} \) is a current estimate of \( x \), \( \hat{x}, \bar{x} \) are estimation error for \( \dot{\hat{x}}, \bar{x} \), respectively, and \( L_i \in \mathbb{R}^{nxp} \) is a \( i \)-th state estimator gain. Then, from the above equations, we can derive the prediction estimation error equation as follows, from which we can easily understand how \( w \) causes an estimation error:

\[
\dot{\hat{z}}(k,0) = x(k,i) - \bar{x}(k,i), \quad \bar{x}(k,i) = \hat{x}(k,i) - \bar{x}(k,i)
\]

(2.4)

Now we add an integrator to the estimation error feedback structure, in order to alleviate the detrimental effect of \( h \) in (1.4), as follows:

\[
\begin{align*}
\bar{x}(k,i+1) &= \Phi w(k,i) + \Gamma u(k,i) \\
\bar{x}(k,0) &= \hat{x}(k,0) \\
\hat{x}(k,i) &= \bar{x}(k,i) + L_i\{y(k,0) - Cx(k,0)\} + \Theta \dot{\hat{x}}(k,0) \\
\dot{\hat{z}}(k,0) &= \bar{x}(k,0) + K\{y(k,0) - Cx(k,0)\}
\end{align*}
\]

(2.5)

where \( z \in \mathbb{R}^l \) denotes an integral state, \( K \in \mathbb{R}^{nxp} \) denotes a corresponding gain matrix, and \( \Theta \in \mathbb{R}^{nxj} \) denotes a weighting matrix that allows us to arbitrarily choose the desired feedback channel. It should be pointed out that this structure has flexibility with respect to the size of integral state, as we have used the notation \( j : 1 \leq j \leq p \) for the definition of the size of each associated matrix and vector.

Then from (1.2) and (2.5), the prediction estimation error dynamics can be described as follows:

\[
\begin{align*}
\hat{\dot{\hat{z}}}(k,0) &= \Phi^r(\hat{\dot{\hat{z}}}(k,0)) + \Gamma \hat{\dot{\hat{z}}}(k,0) \\
\hat{\dot{\hat{z}}}(k,1) &= \Phi^r(\hat{\dot{\hat{z}}}(k,1)) + \Gamma \hat{\dot{\hat{z}}}(k,1) \\
\hat{\dot{\hat{z}}}(k,p) &= \Phi^r(\hat{\dot{\hat{z}}}(k,p)) + \Gamma \hat{\dot{\hat{z}}}(k,p)
\end{align*}
\]

(2.6)

where

\[
\begin{align*}
M(i) &= \Phi(i) - \sum_{j=1}^{i-1} \Phi(i-j) + \Theta \Sigma \hat{\dot{\hat{z}}}(k,0) \\
N(i) &= -\sum_{j=1}^{i} \Phi(i) \hat{\dot{\hat{z}}}(k,0)
\end{align*}
\]

(2.7)

and its characteristics equation is as follows:

\[
\det\left[ \begin{array}{cc} zl - M & -N \\ -KC & zl - I \end{array} \right] = 0
\]

(2.8)

It is of interest to see from (2.6) that, by letting \( \Theta = \Phi^r \Theta \), the uncertainty term \( w \) can be canceled out by the integral state \( z \) through \( N \). In this case, \( x, \bar{x}, \hat{x}, \Theta \in \mathbb{R}^{nxp} \).

On the other hand, regarding the current estimation, the following relation is firstly considered using (2.5):

\[
\begin{align*}
\bar{x}(k,1) &= X^{-1}\hat{\dot{\hat{z}}}(k,1) + X^{-1}\Theta \hat{\dot{\hat{z}}}(k,1) \\
\hat{\dot{\hat{z}}}(k,1) &= X\hat{\dot{\hat{z}}}(k,1) + X\Theta \hat{\dot{\hat{z}}}(k,1)
\end{align*}
\]

(2.9)

(2.10)

By substituting (2.9) and (2.10) for (2.6), we have:

\[
\begin{align*}
\hat{\dot{\hat{z}}}(k,0) &= X^{-1}\hat{\dot{\hat{z}}}(k,1) + X^{-1}\Theta \hat{\dot{\hat{z}}}(k,1) \\
\hat{\dot{\hat{z}}}(k,1) &= X\hat{\dot{\hat{z}}}(k,1) + X\Theta \hat{\dot{\hat{z}}}(k,1)
\end{align*}
\]

(2.11)

Note that we have ignored the uncertainty term here because our aim is to calculate the closed-loop characteristic equation:

\[
\begin{align*}
\det\left[ \begin{array}{cc} zl - \Phi^r + \Theta \Sigma & -R \\ -KC & zl - I - KC \end{array} \right] = 0
\end{align*}
\]

(2.12)

Thus, the characteristic equation for the current estimation error is equivalent to that for the prediction estimation error.

To design the estimator gain, (2.6) can be rewritten as follows:

\[
\begin{align*}
\hat{\dot{\hat{z}}}(k,0) &= \Phi^r(\hat{\dot{\hat{z}}}(k,0)) + \Gamma \hat{\dot{\hat{z}}}(k,0) \\
\hat{\dot{\hat{z}}}(k,1) &= \Phi^r(\hat{\dot{\hat{z}}}(k,1)) + \Gamma \hat{\dot{\hat{z}}}(k,1) \\
\hat{\dot{\hat{z}}}(k,p) &= \Phi^r(\hat{\dot{\hat{z}}}(k,p)) + \Gamma \hat{\dot{\hat{z}}}(k,p)
\end{align*}
\]

(2.13)

Therefore, we can use conventional pole placement method. In addition, we may use the same proportional estimator gain for all \( L_i \) s, which can be calculated by using the design result of single-rate estimator as
follows,
\[
L_{sr} = L_{0} = L_{1} = \cdots = L_{r-1} + \Theta_{r}K
\]  
(2.14)
where \(L_{sr}\) is a single-rate estimator gain. In this case the integral gain, \(K\), of both multi-rate and single-rate are same. Note that it does not necessarily imply it is the best way of selection of multi-rate estimation gain.

3. Discontinuous feedback

For notational convenience, let us now define the matrix \(\Phi_{obs}(i)\) as follows:
\[
\Phi_{obs}(i) = \begin{bmatrix}
\Phi_{obs11}(i) & \Phi_{obs12}(i) \\
\Phi_{obs21}(i) & \Phi_{obs22}(i)
\end{bmatrix}
\]
(3.1)
where \(\Phi_{obs11} \in \mathbb{R}^{m \times m}, \Phi_{obs12} \in \mathbb{R}^{m \times n}, \Phi_{obs21} \in \mathbb{R}^{n \times m}, \Phi_{obs22} \in \mathbb{R}^{n \times n}\).

Instead of using the current estimation update of (2.5), here we propose a multi-rate estimator with a discontinuous structure, which can be described as follows:
\[
\dot{z}(k,i) = T(z(k,i) + L_{r}y(k,0) - CTz(k,0)) + \Theta_{r}\hat{z}(k,0) + \Theta_{v}v(k,0)
\]  
(3.2)
where the discontinuous term is feedback so that states go to the same direction as the uncertainty. This can be easily understood by thinking of a fast varying uncertainty whose frequency is almost the same as Nyquist frequency.

First, the discrete-time switching function is defined as follows:
\[
\nu(k,0) = \begin{cases} 
-\kappa(k,x,u) & \text{if } DC_{x}z_{r} \neq 0 \\
0, & \text{otherwise}
\end{cases}
\]  
(3.3)
where \(\kappa \in \mathbb{R}\) is a design positive scalar function that satisfies \(\kappa \geq h^*\). \(C_{r} = \begin{bmatrix} C & 0 \end{bmatrix}\), and \(D \in \mathbb{R}^{n \times p}\) is some matrix that satisfies the following structural constraint:
\[
C_{r}^TDC_{r} = \Phi_{obs}P_{r}\Theta_{r},
\]
\[
\Theta_{r}^T = [Y_{r}, 0], \quad Y_{r} = \Omega^{-1}\sum_{j=1}^{r} \Phi_{j}^{T}
\]  
(3.4)
where \(P_{r}\) is a positive definite matrix which is the unique solution to the following Lyapunov function:
\[
\Phi_{obs}P_{r}\Phi_{obs} - P_{r} = -Q_{r}
\]  
(3.5)
where \(Q_{r} \in \mathbb{R}^{(n+q) \times (n+q)}\) is a symmetric positive definite design matrix.

Regarding this structural constraint, it should be pointed out that, for a single output HDD, \(D\) in (3.4) can be set to one. It is because \(p = 1\) implies \(q = 1\) from the dimensional restriction \(q \leq p\), and \(D \in \mathbb{R}\). Thus, it is obvious from (3.3) that the matrix \(D\) can be set to one. Further discussion related to the selection of \(D\) in (3.4) for MIMO system can be found in [7].

**Theorem 1**: Suppose that \(L\) and \(K\) are designed such that \(\rho(\Phi_{obs}) \leq 1\). Then there always exist an ultimately bounded sliding mode observer.

**Proof**: By taking a coordinate transformation, \(e_{z}^{T} = \begin{bmatrix} e_{z} - X_{\Theta}h \end{bmatrix}\), and also adding the discontinuous and uncertainty terms, (2.11) can be rewritten as follows:
\[
e_{za}(k+1) = \Phi_{obs}e_{za}(k) + \Theta_{r}\hat{z}(k,0) + \Theta_{v}v(k)
\]  
(3.6)
Now define the Lyapunov candidate function:
\[
V_{obs}(k) = e_{za}^{T}P_{z}e_{za}
\]  
(3.7)
and evaluate along the trajectories of (3.6), we have:
\[
V_{obs}(k+1) - V_{obs}(k) < -\lambda_{min}(Q_{z})\|e_{za}\|^2 + \|\Theta_{r}P_{z}\|(h^* + \kappa)^2
\]  
(3.8)
Since \(Q_{z} \) is positive definite, it is upper convex with respect to \(e_{za}\) and there always exists ultimately bounded sliding mode. The size of boundary can be obtained by solving above equation with respect to \(\|e_{za}\|\) as follows:
\[
\|e_{za}\| < \sqrt{\frac{\lambda_{min}(Q_{z})}{\lambda_{max}(Q_{z})}} (h^* + \kappa)
\]  
(3.9)
Physical meaning of above boundary is that, if the signum of switching function is different from that of the uncertainty, the discontinuous term is feedback so that states go to the same direction as the uncertainty. This can be easily understood by thinking of a fast varying uncertainty whose frequency is almost the same as Nyquist frequency.  

In practical design, for this reason, we may apply a band-limit filter to \(\kappa\) of (3.3). In addition, we may introduce the saturation function or other kind of function so as to alleviate the chattering motion.

4. Equivalent linear state feedback

Although we may invoke a conventional design method for designing a state feedback gain as a regulator, here we derive a sliding mode based linear multi-rate state feedback gain.

First, the discrete-time switching function is defined with a reference input \(x_{r} \in \mathbb{R}\) as follows:
\[
S(k,i) = G(x(k,i) + x_{r}(k,i))
\]  
(4.1)
where \(G \in \mathbb{R}^{m \times n}\) is a sliding hyperplane matrix. Usually \(G\) is designed to be a static matrix, which means the control bandwidth is not satisfactorily taken into consideration and it may lead to a so-called
spillover problem. For this reason, several methods of
designing the dynamic hyperplane so that it has some
dynamics in itself have been exploited [11] in an attempt
to reduce the spillover problem. This concept works
well. In this paper, an alternate simple dynamic structure
is derived without augmenting the state space. To do so,
the following condition is considered:
\[ S(k+1) = \alpha S(k) \]
\[ \alpha = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_n), 0 \leq \alpha_1, \alpha_2, \ldots, \alpha_n < 1 \]  
(4.2)
where \( S \) was defined in (4.1). The introduction of the
parameter \( \alpha \) is to reduce the spillover problem as
discussed above. In this case, sliding hyperplane itself
does not have dynamics, but the switching function
can be thought of as having a first order dynamics without
the need to introduce an additional state. Therefore, \( \alpha \)
can be designed as follows:
\[ \alpha = \exp(-2\pi f_jT_s) \]
(3.3)
where \( f_j \) denotes the cut-off frequency of \( j \)-th element
of switching function (or control input).

The equivalent control input that holds the states
onto the settling sliding surface can be obtained by
letting:
\[ S(k, i+1) = \alpha S(k, i) \]
\[ Gx(k, i+1) + G\dot{x}_k(k, i+1) = \alpha Gx(k, i) + \alpha G\dot{x}_k(k, i) \]
\[ G[\Phi_m x(k, i) + \Gamma_m u(k, i)] + G\dot{x}_k(k, i+1) = \alpha Gx(k, i) + \alpha G\dot{x}_k(k, i) \]
(4.4)

From (4.4), we obtain the following equivalent control
input:
\[ u_{eq}(k, i) = -(G\Gamma_m^{-1}.(G\Phi_m - \alpha G))\dot{x}(k, i) \]
\[ - (G\Gamma_m^{-1} \{ G\dot{x}_k(k, i+1) - \alpha G\dot{x}_k(k, i) \} \]
(4.5)

Note that the current state estimator is used in the
above equation.

Regarding the equivalent dynamics of multi-rate
system, by substituting \( u \) of (1.2) for (4.5) as well as
letting \( w \) of (1.2) and \( x \) of (4.5) be 0, we have the following
equation:
\[ x(k, i+1) = \Phi_{eq} x(k, 0) - \sum_{j=0}^{i} \Phi_{eq}^{-1}\Gamma_{eq} K_{eq} \hat{e}(k, j) \]
(4.6)

In the above equation, the following matrix was
defined:
\[ \Phi_{eq} \triangleq \left( \Phi_m + \Gamma_m K_{eq} \right), \Phi_{eq} \in \mathbb{R}^{n_{eq}} \]  
(4.9)
By substituting the estimation error in (4.8) for
(2.11), we have:
\[ x(k+1, 0) = \Phi_{eq} x(k, 0) \]
\[ - \sum_{j=0}^{i-1} \Phi_{eq}^{-1}\Gamma_{eq} K_{eq} \Phi_{obs11}(j) \hat{e}(k, 0) \]
(4.10)

Therefore, the considered dynamics of (2.11) and
(4.10) can be written as follows:
\[ \begin{bmatrix} \hat{e}(k+1, 0) \\ \hat{z}(k, 0) \\ x(k+1, 0) \end{bmatrix} = \begin{bmatrix} \Phi_{obs} & 0 \\ E & \Phi\tau_{eq} \end{bmatrix} \begin{bmatrix} \hat{e}(k, 0) \\ \hat{z}(k-1, 0) \\ x(k, 0) \end{bmatrix} \]
(4.11)
\[ E \in \mathbb{R}^{n_{eq}+p} \]

This eigenstructure shows that \( G \) and \( L \) can be
designed separately. Note that \( G \) can be designed such
that \( GT \) is nonsingular without any serious difficulty
since \( B \) is full rank by assumption. Also it should be
pointed out that \( G \) and \( \alpha \) can be designed separately.

**Theorem 2:** Suppose that \( G \) is designed such that
\( \rho(\Phi_m - \alpha G, (G\Gamma_m^{-1} \{ G\dot{x}_k(k, i+1) - \alpha G\dot{x}_k(k, i) \}) < 1 \). Then there always exist an
asymptotically stable system for parameter
\( 0 < \alpha < 1 \) such that \( \rho(\Phi_{eq}) < 1 \). In this case, \( G \) and \( \alpha \)
can be designed separately.

**Proof:** The system can be transformed into a canonical
form using some coordinate transformation matrix
defined as follows:
\[ T = \begin{bmatrix} I_{n_{eq}+n_{au}} & O_{n_{eq}+n_{au}} \\ G_1 & G_2 \end{bmatrix} \]
(4.12)
where \( I \) denotes the identity matrix, and \( G = [G_1, G_2] \).

Note that the system is pre-transformed into the
controllable canonical form before applying \( T \) such
that the system and control distribution matrix have the form:
\[ \Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}, \Gamma = \begin{bmatrix} O \\ \Gamma_2 \end{bmatrix} \]
(4.13)

Then, (4.9) can be transformed to:
\[ T\Phi_{eq} T^{-1} = \begin{bmatrix} \Phi_{11} - \Phi_{12} G_2^{-1} G_1 & \Phi_{12} G_2^{-1} O \\ \Phi_{21} & \alpha \end{bmatrix} \]
(4.14)

The eigenvalues of equivalent system matrix consist of
that of \( \Phi_{11} - \Phi_{12} G_2^{-1} G_1 \) which is designed to be
stable by the hyperplane matrix \( G \), and \( \alpha \) of which
each element is less than one. Therefore the equivalent
system is stable for \( 0 < \alpha_j < 1 \), and \( G \) and \( \alpha \) can be
designed separately.

The hyperplane matrix \( G \) can be thought of as a
state feedback gain, and it can be determined in the \( H_2 \)
sense in order to minimize the error from the ideal
sliding hyperplane by solving the following discrete
algebraic Riccati equation [12]:
\[ \Phi^T \Phi \sigma - P - \Phi^T \Phi T_D^{2/2} T_D^{1/2} + W = 0 \] (4.15)

where \( P \) is the unique solution of (4.15), \( W \) is a weighting, and \( \Phi \sigma \) is a modified system matrix [13] with stability margin \( \varepsilon \) defined as follows:
\[ \Phi \sigma = \Phi m + \varepsilon I, 0 \leq \varepsilon < 1 \] (4.16)

With this modified system matrix associated with \( \varepsilon \), we can design \( G \) such that the spectral radius of the closed-loop system other than \( \alpha \) be less than \( 1 - \varepsilon \). This implies that the cutoff frequency of associated modes can be described with \( \varepsilon \) as follows:
\[ f_c \geq -\frac{\log (1 - \varepsilon)}{2\pi T_m} \] (4.17)

5. Design Example and Experiments

First, we consider a double integrator model for a VCM in a single stage actuated HDD. Table 1 shows the design parameters.

Table 1: Design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling Freq</td>
<td>1/T _a</td>
</tr>
<tr>
<td>Control updating Frequency</td>
<td>1/T _w</td>
</tr>
<tr>
<td>Multi-rate ratio</td>
<td>r</td>
</tr>
<tr>
<td>VCM gain</td>
<td>K _s</td>
</tr>
</tbody>
</table>

Multi-rate equivalent model can be described as follows:
\[
\Phi = \begin{bmatrix} 1 & T_m \\ 0 & 1 \end{bmatrix}, \Gamma = K_f \begin{bmatrix} T_m^2/2 \\ T_m \end{bmatrix}, C = [1 \ 0] \] (5.1)

Here we invoke Ackerman’s formula [14] to place the poles at:
\[
p = [0.2593+0.3192i \ 0.2593-0.3192i \ 0.9590] \] (5.2)

Then, from (2.13), we have:
\[
L_m = [3.5287e-1 \ 3.3191e3], K = 6.0645e2 \] (5.3)

Regarding the plant uncertainty, we consider here a VCM gain perturbation of 30%. Thus, the discontinuous term in (3.3) can be described as follows:
\[
\nu(k,0) = -0.3 |\delta| \frac{C(x(k,0) - y(k,0))}{|C(x(k,0) - y(k,0))| + \delta} \] (5.4)

where \( \delta \) is a design parameter for alleviating the undesired chattering motion.

For the regulator, we let \( f \) in (4.3), \( f_c \) in (4.17), and \( W \) in (4.15) be 1kHz, 1kHz, and \( \text{diag}(1,1) \) , respectively, and obtain the following hyperplane matrix:
\[
G = [2.7920e4 \ 3.4652] \] (5.5)

Moreover, we applied a series of conventional second order notch filters to compensate for VCM resonant modes at around 6kHz and 8kHz.

Using these design parameters, we conducted an experiment on a 3.5 inch 7200rpm low profile HDD driven by a floating point DSP (TMS320C6711) equipped with 14bit ADC, 12bit DAC.

In figure 1, we compared the typical sinusoidal response to a 500Hz reference signal for conventional controller against that for our proposed controller. As it can be seen from this figure, with the conventional controller, the estimation error cannot be ignored when the VCM gain is perturbed. Our proposed controller achieved almost perfect tracking compared with the conventional one, even under such conditions, and estimation error is suppressed to the same level as the nominal case as shown in figure 2.

![Figure 1: Sinusoidal Response at 500Hz](image1)

![Figure 2: Comparison of Estimation Error](image2)

We then conducted an eight tracks seeking experiment on the same HDD mentioned above, and compared the result for conventional controller against the controller proposed here.

Figure 3 and 4 show the forward and backward seeking responses (five times each) for the conventional and proposed controllers with -30% perturbation of VCM actuator gain. Our proposed method achieves a 1ms seek time, while conventional controller takes almost 2.5ms because of the presence of residual vibrations. In this experiment, we only used the SMART [15] trajectory as a reference input and no other method was applied.

We also carried out a seeking control experiment for a dual stage actuated HDD, which is the same HDD mentioned above but has a PZT actuator on tip of the arm, and have confirmed the similar results. The best achievable seeking time for the SMART trajectory...
driven one track seeking was 0.2ms.

6. Conclusion
A new multi-rate state estimator for seeking control of hard disk drives, which has a proportional, integral, and discontinuous estimation error feedback structure, was proposed to improve controller robustness against disturbance inputs and model parametric variation. The proposed controller was validated by carrying out some experiments. Simulation and experimental results confirmed that the robustness of the feedback system against actuator gain perturbation was improved as compared to conventional servoing method.

Reference